# Analyses of loci of bamboomerangs and their educational effects

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ABSTRACT: The Department of Mechanical Engineering at Tokyo Metropolitan College of Aeronautical Engineering has many laboratories. In the authors' laboratory, developments, experiments and analyses of various children's toys are carried out with students. For example, boomerangs with three wings, PET Bottle Rockets, boomerang stunt plains, flying rings and boomerang paper cups are developed and analysed by simple equations of motion. The analytical results can explain the experiments. A bamboomerang is a boomerang dragonfly made of bamboo in Japan. A bamboomerang has a small weight at both wing ends and a spindle. It is launched on an angle. When it flies, it goes up and returns to the player's hands. This movement looks like a pendulum motion. The loci of bamboomerang were analysed by simple equations of motion. Analysed results coincide with other experiments. The authors taught school children how to use the bamboomerang. Based on the responses from them, it seems that the bamboomerangs are good technology education teaching materials for school children.

## INTRODUCTION

A bamboomerang, as shown in Figure 1, is a boomerang dragonfly made of bamboo in Japan. It has a slight of weight at both ends and a spindle at the end. It is launched with a slight of slant. When it flies remote, it goes up and returns to the payer's hands. This movement looks like a pendulum motion. The loci of bamboomeransgs are analysed by simple equations of motion.

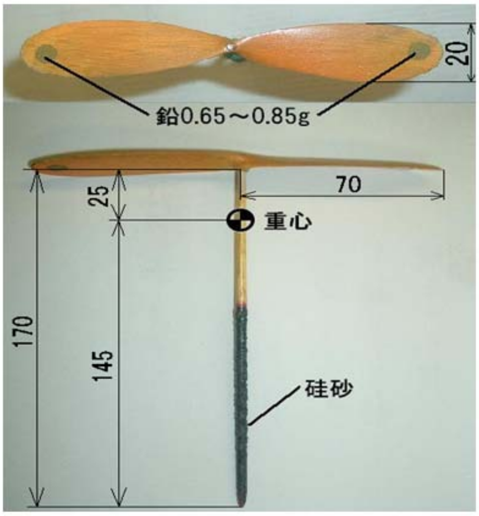


Figure 1: A bamboomerang made of bamboo.

## EQUATIONS OF MOTION

Figure 2 shows the frame of reference of this analysis. Lift $L$ acts on the travelling direction of the bamboomerang. Drag $D$ acts on the direction where the flight is hindered. Gravity $mg$ acted on the perpendicular downward from the centre of gravity. Bamboomerang flies at an initial angle $\theta_0$ and an initial velocity $v_0$. Equations of motion of the direction of $x$ and $z$ axes and the angle $\theta$ becomes:

$$m\frac{d^2x}{dt^2} = L\sin\theta - D \tag{1}$$

$$m\frac{d^2z}{dt^2} = L\cos\theta - mg \tag{2}$$

$$J\frac{d^2\theta}{dt^2} = -mg\ell\sin\theta \tag{3}$$

where, *m* is the weight of the bamboomerang, *J* is a moment of inertia around the centre of gravity and *l* is the distance from the centre of the wing to the centre of gravity. Moreover, in this analysis, it was assumed that there was a relationship:

$$L = L_o e^{-\varepsilon t} \tag{4}$$

where, *t* is the flight time, $L_0$ is the lift at the time of flight start and $\varepsilon$ is a constant.

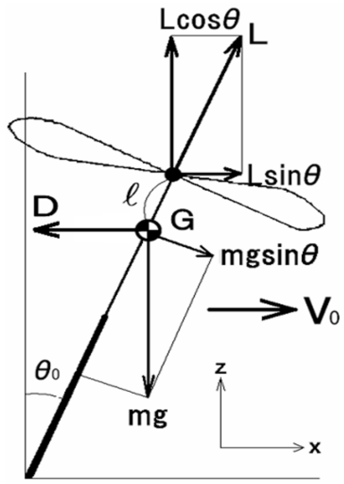


Figure 2: Coordinate axis.

ANALYSES OF LOCI

First, when $\theta$ is assumed to be a minute, it becomes $\sin\theta=\theta$. Therefore, Equation (3) becomes:

$$\frac{d^2\theta}{dt^2} + \omega_n{}^2\theta = 0 \tag{5}$$

where, $\omega_n = \sqrt{\frac{mg\ell}{J}}$.

When Equation (5) is solved under the initial condition, $t = 0, \theta = \theta_0, \dot{\theta} = 0$, it becomes:

$$\theta = \theta_0 \cos\omega_n t \tag{6}$$

Second, solves Equation (1), when $\theta$ is assumed to be a minute, as well as Equation (3), and Equation (4) and Equation (6) are substituted; Equation (1) becomes:

$$\frac{d^2x}{dt^2} = \frac{L_0\theta_0}{m} e^{-\varepsilon t} \cos\omega_n t - \frac{D}{m} \tag{7}$$

General solution $x_n$ and particular solution $x_s$ of homogeneous equation of Equation (7) is:

$$x_n = \frac{C_1}{m}t + \frac{C_2}{m} \tag{8}$$

$$x_s = Ae^{-\varepsilon t}\cos\omega_n t + Be^{-\varepsilon t}\sin\omega_n t + Ct^2 \tag{9}$$

where $C_1$, $C_2$, $A$, $B$ and $C$ is constant. When Equation (9) is differentiated twice, and substituted into Equation (7), it is then obtained:

$$A = \frac{\alpha L_0 \theta_0}{m(\alpha^2 + \beta^2)},\ B = -\frac{\beta L_0 \theta_0}{m(\alpha^2 + \beta^2)},\ C = -\frac{D}{2m}.$$

where $\alpha = \varepsilon^2 - \omega_n^2,\ \ \beta = 2\varepsilon\omega_n$. Therefore, when it assumed to be $\frac{L_0\theta_0}{m(\alpha^2+\beta^2)} = \gamma$, because $x_s$ is:

$$x = \alpha\gamma e^{-\varepsilon t}\cos\omega_n t - \beta\gamma e^{-\varepsilon t}\sin\omega_n t - \frac{D}{2m}t^2 \tag{10}$$

the entire general solution is:

$$x = \frac{C_1}{m}t + \frac{C_2}{m} + \alpha\gamma e^{-\varepsilon t}\cos\omega_n t - \beta\gamma e^{-\varepsilon t}\sin\omega_n t - \frac{D}{2m}t^2 \tag{11}$$

When Equation (11) is differentiated, and the initial condition substituted, $t$=0, $x$=0, $\dot{x} = v_0$, into it, the final general solution is:

$$x = \{v_0 + (\varepsilon\alpha + \omega_n\beta)\gamma\}t + \gamma(\alpha\cos\omega_n t - \beta\sin\omega_n t)e^{-\varepsilon t} - \frac{D}{2m}t^2 - \alpha\gamma \tag{12}$$

Finally, solves Equation (2), when $\theta$ is assumed to be a minute, as well as Equation (3) and Equation (4) are substituted, Equation (2) becomes:

$$\frac{d^2 z}{dt^2} = \frac{L_0}{m}e^{-\varepsilon t} - g \tag{13}$$

The general solution $z_n$ and particular solution $z_s$ of homogeneous equation of Equation (13) is:

$$z_n = \frac{C_3}{m}t + \frac{C_4}{m} \tag{14}$$

$$z_s = Ee^{-\varepsilon t} + Ft^2 \tag{15}$$

where $C3$, $C4$, $E$ and $F$ is constant. When Equation (15) is differentiated twice, and substituted into Equation (13), it becomes:

$$E = \frac{L_0}{m\varepsilon^2},\ F = -\frac{g}{2}.$$

Therefore, the entire general solution is:

$$z = \frac{C_3}{m}t + \frac{C_4}{m} + \frac{L_0}{m\varepsilon^2}e^{-\varepsilon t} - \frac{g}{2}t^2 \tag{16}$$

When Equation (16) is differentiated, and the initial condition substituted, $t$=0, $z$=0, $\dot{z} = 0$, into it, the final general solution is:

$$z = \frac{L_0}{m\varepsilon}t + \frac{L_0}{m\varepsilon^2}(e^{-\varepsilon t} - 1) - \frac{g}{2}t^2 \tag{17}$$

## COMPARISON BETWEEN THE ANALYTICAL RESULTS AND THE EXPERIMENT

Lift $L$ and drag $D$ are shown by next equations:

$$L = 1/2C_L \rho V^2 S, D = 1/2C_D \rho V^2 S \quad (18)$$

When $C_L$=1.2, $C_D$=0.17, $\rho$=1.2kg/m$^3$, $V$=9m/s, $S$=0.0014m$^3$ are subsutituted for Equation (18), $L$=0.08N, $D$=0.0116N are the results.

When $m$=0.007kg, $v_0$=3m/s, $\theta_0$=10°, $L$=0.08N, $D$=0.0116N, $\omega_n$=9rad/s are substituted for theoretical equations, Figure 3 is obtained. Figure 4 shows the real locus. It was found that the theoretical locus agreed with the real locus.

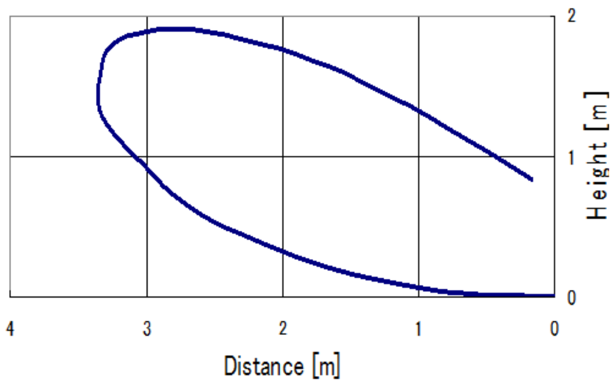


Figure 3: Analytical locus.

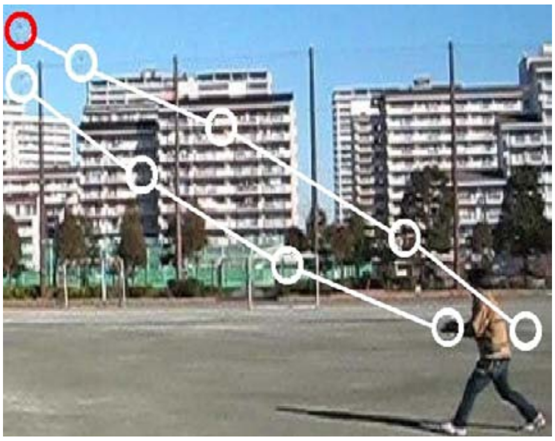

Figure 4: Real locus.

EDUCATIONAL EFFECTS - SCHOOL CHILDREN IMPRESSIONS

The authors and some students taught the school children (with their parents) how to use the bamboomerang. Their impressions are summarised as follows:

- *I was very interested. I experienced surprise from using familiar materials.*
- *I was surprised by the returning bamboomerang.*
- *I think it provides a good experience for children.*
- *I think children like science and engineering.*
- *My parents are getting very interested in my manufacturing and playing with the bamboomerang.*
- *I could feel refreshed by making and playing with the bamboomerang.*

CONCLUSIONS

The analyses of loci of bamboomerangs and impressions of school children are introduced and presented. The authors intend to demonstrate various types of boomerangs, including bamboomerangs during their paper presentation at the Conference. It seems that the bamboomerangs are good teaching materials for technology education to school children.