

Formulating structural matrices of equations of electric circuits containing dependent sources, with the use of the Maple program

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ABSTRACT: The method of deriving loop matrix impedances BZB^T and the loop voltage vector B(E-ZJ) in electric linear circuits containing dependent sources, is described in this paper. Primary equations of the electric circuit were formulated, using the classical method, for the loop current method of network analysis. The transformation of them was derived within the mathematics software program, Maple. The method of analysis of the electric circuit presented in this paper was illustrated with an example.

INTRODUCTION

Circuit theory is a fundamental engineering discipline that pervades all electrical engineering, therefore learning electric circuit theory in electrical engineering education, is important. The goal of circuit theory is to make quantitative and qualitative calculations of the electrical behaviour of circuits [3]. For electrical engineers not only is it important to know elementary circuit theory, it is also important to understand the methods for handling more complex circuit theory.

Students who specialise in electrical engineering or electronics typically need to study not only DC and AC circuits with techniques such as Kirchhoff's voltage law, Kirchhoff's current law and Ohm's law, but also topology methods with matrix for handling more complex networks. They are usually required to learn how to apply various transform methods (phasor, Fourier and Laplace) and Fourier-series, in circuit analysis. Understanding concepts from circuit theory, especially DC and AC electricity, periodic signals and transients, is essential to understanding subjects such as electronics, telecommunication, power engineering, system theory and automated technology.

Studies into the teaching of electricity and circuit theory have shown that students, even at university or college level, have difficulties in acquiring a functional understanding of, and distinguishing between, fundamental concepts such as current, voltage, energy and power. Students' lack of qualitative understanding is the reason why difficulties arise in correctly solving quantitative problems [1]. Insufficient mathematical knowledge is often a barrier for students in the understanding of the theory of circuits. The way one thinks about mathematics in circuit theory differs from what is taught in the subject of mathematics. This process is not transparent for students. Mathematical computer programs can be helpful as a solution to this problem [4][5].

NETWORK ANALYSIS

The goal of network analysis is to determine the voltages and currents associated with the elements of an electrical network so as to predict the electrical behaviour of real physical circuits. The purpose of these predictions is to improve their design; particularly to decrease their cost and improve their performance under all conditions of operation. Kirchhoff laws are fundamental postulates of circuit theory. They are valid regardless of the nature of circuit elements. Therefore, the separation between the equations of Kirchhoff's laws and that of elements' characteristics is natural.

Kirchhoff's equations are prescribed by the topology of the circuit, that is, the way circuit elements are interconnected. The element characteristic is prescribed by the voltage-current (v-i) relations given by the laws of physics. However, these equations involve a large quantity of variables. Using graph theory diminishes the number of unknowns. The theory of graphs plays a fundamental role in exploring the structural properties of electrical circuits [7].

The graphs are good pictorial representations of circuits and capture all their structural characteristics. In circuit analysis using graph theory, there is compliance with three transformations: loop transformations, cutest transformations and node transformations. Only the method of analysis of electric circuits based on loop transformation is applied in this paper. Given solutions can be applied in the manner of methods of analysis based on extant transformations.

LOOP TRANSFORMATIONS AND LOOP SYSTEM OF EQUATIONS

An electrical circuit is an interconnection of electrical network elements, such as resistances, capacitances, inductances, independent and dependent voltages and current sources. Each network element is associated with two variables: the voltage variable $v(t)$ and the current variable $i(t)$.

The electrical network N can be represented by corresponding with its directed graph G. Let T be a spanning tree of an electrical network. Let I_{c} be the column vectors of chord currents and I_{b} be the column vectors of branch currents with respect to T. The submatrix of circuit matrix B_c corresponding to the fundamental circuits defined by the chords of a spanning tree T is called fundamental circuit matrix B_f of G with respect to the spanning tree T. The equation below presents the loop transformation.

$$I_{\mathrm{b}} = B_{\mathrm{f}}^{\mathrm{T}} I_{\mathrm{c}} \tag{1}$$

As can be seen from the loop transformations, not all these variables are independent. Furthermore, in place of Kirchhoff's voltage law equations, the loop transformation, which involves only chord currents as variables, can be used. Kirchhoff's voltage law is given below:

$$B_{\mathrm{f}} V_{\mathrm{b}} = 0 \tag{2}$$

The voltage-current relations for every branch of the electric network are:

$$V_{\mathrm{b}} = Z_{\mathrm{N}} I_{\mathrm{b}} \tag{3}$$

where Z_N is the branch impedance matrix.

The advantage of these transformations is that it enables different systems of network equations known as loop systems to be established. In deriving the loop system, the loop transformation is used, and loop variables (chord currents) serve as independent variables. Partition of the chord currents vector I_{c} and element voltage vector V_{b} is:

$$I_{\mathrm{c}} = \begin{bmatrix} I_{\mathrm{c\_nsc}} \\ J \end{bmatrix} \quad \text{and} \quad V_{\mathrm{b}} = \begin{bmatrix} V_{\mathrm{b\_nsv}} \\ E \end{bmatrix} \tag{4}$$

where $I_{\mathrm{c\_nsc}}$ is the vector of currents in the non-current source chords of G, J is the current sources vector, $V_{\mathrm{b\_nsv}}$ is the vector of branches voltage in the non-voltage source and E is the voltage sources vector.

Using the above-dependencies, this equation is derived:

$$B_{\mathrm{f}} Z_{\mathrm{N}} B_{\mathrm{f}}^{\mathrm{T}} I_{c\_ns} = B_{\mathrm{f}} \left(E - Z_{\mathrm{N}} J \right) \tag{5}$$

Equation 5 is called the loop system of equations and matrix $B_{\mathrm{f}} Z_{\mathrm{N}} B_{\mathrm{f}}^{\mathrm{T}}$ is the loop impedance matrix.

Since presenting examples is a fundamental element of the didactic process, a computational example is introduced below that illustrates the problems above. The formulation of structural matrices of the electric circuit, with the help of the Maple software program, is given below.

THE MATHEMATICAL PROGRAM, MAPLE

There are a few very good commercial programmer environments for mathematical computations and one of them is Maple. Maple™ is useful technical computing software for engineers, mathematicians, scientists and educators [2][6].

Taking into consideration a large quantity of literature and, especially, the availability of free examples on the Internet, the authors of this paper decided to use this program to solve problems connected with circuit theory. Maple is a complete mathematical problem-solving environment that supports a wide variety of mathematical operations, such as numerical analysis, symbolic algebra and graphics. It allows a choice of more than 4,000 commands ranging from performing basic arithmetic and algebra, to computations involving advanced topics such as tensor analysis, group theory, and more. Maple offers interactive mathematical visualisation, a user interface with typeset mathematics, word processing facilities, and a modern programming language, making it powerful and flexible for users in education, research and industry. Maple is a large and complicated program. But despite that, thanks to the examples, it is relatively easy to use to solve different assignments in the field of circuit theory. Using Maple makes it possible to work in interactive mode.

To carry out calculations, develop design sheets, teach fundamental concepts or create complicated simulation models, Maple's computation engine can handle every type of mathematics. With Maple, interactive documents can be created. The software has two modes: Document mode and Worksheet mode. Document mode is designed for performing calculations quickly.

Worksheet mode is designed for interactive use through commands and programming using the Maple language. The example below was derived by exploiting the Worksheet mode using Text mode (1-D Math).

ILLUSTRATIVE EXAMPLE

Shown below in Figure 1 is the variety of symbols of the graphic elements of the electric circuit.

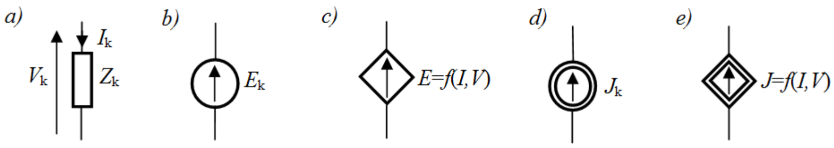

Figure 1: a) Impedance Z_k with current I_k and voltage V_k, b) Independent voltage source, c) Dependent voltage source, d) Independent current source, e) Dependent current source.

Now consider linear circuit impedances, as well as independent and dependent voltage source and current sources. The schema of the electrical network *N* and graph *G* representation of *N* are shown in Figure 1. Branch circuits correspond to graph edges.

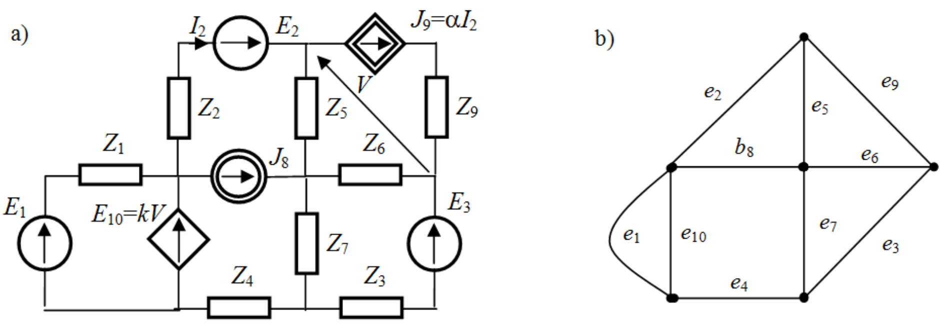

Figure 2: a) The electrical network *N*, b) Graph *G* representation of *N*.

Consider also the connected graph and its spanning tree. A sub-graph of *G* is a spanning tree of *G* if the sub-graph is a tree and contains all the vertices of *G*. The spanning tree of the graph of Figure 2b) is shown in Figure 3b). Edges of a spanning tree *T* are called the branches of *T*. The branches of the tree are marked with double lines. For the given spanning tree of connected graph *G*, the co-spanning tree relative to *T* is the sub-graph of *G* induced by the edges that are not present in *T*. The edges of a co-spanning tree are called chords. The co-spanning tree relative to the spanning tree *T* of Figure 3b) consists of these chords: c_1, c_2, c_6, c_8, c_9. The spanning tree *T* was chosen so as to make voltages and currents - which control dependent voltage sources and current - to be in chords.

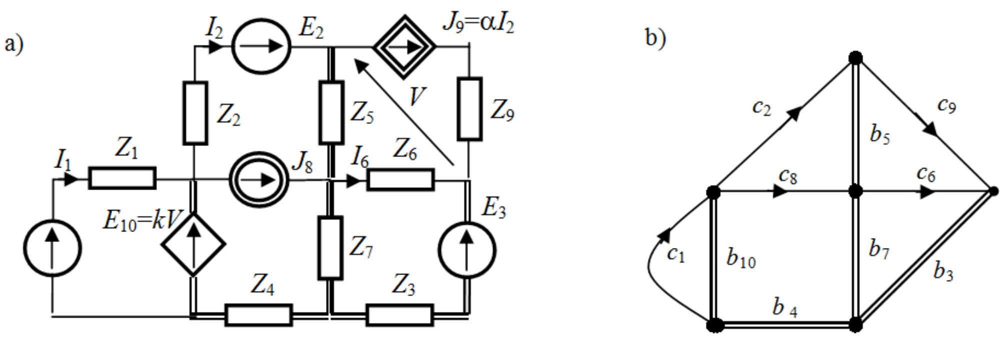

Figure 3: The electrical network *N* and its graph with a spanning tree and directed co-spanning tree.

Because only *current* chords are of interest to the method discussed here, only those signals were marked. The chord currents vector *I*c is as follows:

$$I_c = \begin{bmatrix} I_{c\_nsc} \\ J \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_6 \\ J_8 \\ J_9 \end{bmatrix} \tag{6}$$

Wanted unknown chord currents are:

$$I_{c\_nsc} = [I_1 \quad I_2 \quad I_6]^T \tag{7}$$

The voltages and currents which control dependent sources are expressed respectively by means of chord currents and independent voltages and *current* sources. In the case of the electric circuit here, there is:

$$V = Z_5 I_5 + Z_6 I_6 \tag{8}$$

Because the currents of the branches of the spanning tree are linear, the (8) equation can be changed to:

$$V = Z_5 (I_2 - \alpha I_2) + Z_6 I_6 \tag{9}$$

Shown below is how Maple is used to perform mathematical transformations. The `linalg` packages (Maple library) contain functions that help users to work with matrix and linear system equations. An equation can be created in Maple by any method which leads to the construction of algebraic (or any other) equations.

The restart command causes the Maple kernel to clear internal memory so that it then acts (almost) as if it has just begun.

```
> restart;with(linalg):
```

Equations of the relations of voltage-current for dependent sources.

```
> E[10]:=k*V;
```

$$E_{10} := k\, V$$

```
> J[9]:=alpha*Ic[2];
```

$$J_9 := \alpha\, Ic_2$$

```
> V:=Z[5]*(Ic[2]-J[9])-Z[6]*Ic[6];
```

$$V := Z_5\, (Ic_2 - \alpha\, Ic_2) - Z_6\, Ic_6$$

Kirchhoff's voltage law equation around loop with chord c_1 is written as:

```
> KVL1:=Z[1]*Ic[1] = E[1]-k*V;
```

$$KVL1 := Z_1\, Ic_1 = E_1 - k\, (Z_5\, (Ic_2 - \alpha\, Ic_2) - Z_6\, Ic_6)$$

Kirchhoff's voltage law equation around loop with chord c_2 is written as:

```
> KVL2:=(Z[2]+Z[4]+Z[5]+Z[7])*Ic[2]+Z[7]*Ic[6]+(Z[4]+Z[7])*J[8]-(Z[5]+Z[7])*J[9] =E2+k*V;
```

$$KVL2 := (Z_2 + Z_4 + Z_5 + Z_7)\, Ic_2 + Z_7\, Ic_6 + (Z_4 + Z_7)\, J_8 - (Z_5 + Z_7)\, \alpha\, Ic_2 = E2 + k\, (Z_5\, (Ic_2 - \alpha\, Ic_2) - Z_6\, Ic_6)$$

Kirchhoff's voltage law equation around loop with chord c_6 is written as:

```
> KVL3:=Z[7]*Ic[2]+(Z[3]+Z[7]+Z[6])*Ic[6]-Z[7]*J[8]+(Z[3]+Z[7])*J[9] = -E[3];
```

$$KVL3 := Z_7\, Ic_2 + (Z_3 + Z_7 + Z_6)\, Ic_6 - Z_7\, J_8 + (Z_3 + Z_7)\, \alpha\, Ic_2 = -E_3$$

Loop system of equations:

```
> set_eqn:= [KVL1,KVL2,KVL3]:
```

Determining loop impedance matrix $B_f Z_N B_f^T$ and matrix $B_f\left(E - Z_N J\right)$ uses the following:

```
> BZBT:=genmatrix(set_eqn,[Ic[1],Ic[2],Ic[6]],'b');
```

$$BZBT := \begin{bmatrix} Z_1 & k\,Z_5\,(1-\alpha) & -k\,Z_6 \\ 0 & -(Z_5+Z_7)\,\alpha + Z_2 + Z_4 + Z_5 + Z_7 - k\,Z_5\,(1-\alpha) & Z_7 + k\,Z_6 \\ 0 & (Z_3+Z_7)\,\alpha + Z_7 & Z_3 + Z_7 + Z_6 \end{bmatrix}$$

```
> B:=eval(b):BE_ZJ:=matrix(3,1,B);Icns:=matrix(3,1,[Ic[1],Ic[2],Ic[6]]);
```

$$BE\_ZJ := \begin{bmatrix} E_1 \\ E2 - (Z_4+Z_7)\,J_8 \\ -E_3 + Z_7\,J_8 \end{bmatrix} \qquad Icns := \begin{bmatrix} Ic_1 \\ Ic_2 \\ Ic_6 \end{bmatrix}$$

Then, explore the properties of the loop impedance matrix and find vector of currents I_{c_nsc} in the non-source chords. Remaining currents and voltages in the electric circuit can be calculated from equations (1) and (3).

CONCLUSIONS

The authors have shown in this paper how the mathematics software program, Maple, can be used effectively to formulate structural matrices of the loop system of electric circuit equations. Without performing many intermediate computations, and with the help of Maple, the loop impedance matrix for linear circuits was found. Similarly, using the Maple program, an *elegant* system of equations and node equations can be formulated.

The Maple code demonstrated above is relatively simple. It is easy to draw and animate the solution, too. The symbolic, numerical and graphical features of mathematical programs allow students to carry out mathematical analysis of circuits without time-consuming calculations and to explore easily the behaviour of a system.

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