

Technology education through returnable toys

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ABSTRACT: The Department of Mechanical Engineering at Tokyo Metropolitan College of Aeronautical Engineering consists of several research and teaching laboratories. In the authors' laboratory, new experiments and experimental procedures are being developed, analysed and tested, including various children's toys in which students play a part. The authors have developed and analysed *returning* (returnable) toys for children. In this paper, returnable toys developed in the Department, such as paper boomerangs, boomerang stunt planes, bamboomerangs and boomerang paper cups, are discussed. These analyses are being used to educate students at the College of Technology. It has been proven that returnable flying toys are most effective in technology education because of their simplicity and predictable behaviour.

INTRODUCTION

The authors developed, analysed and tested various types of children's toys and in the process involved students at the Tokyo Metropolitan College of Aeronautical Engineering. These *returnable toys* included paper flying-ring two-legged toys, and a PET rocket. Paper boomerangs, boomerang planes, bamboomerangs and boomerang paper cups are introduced and analysed in this paper. The analyses enable teachers to use the toys as a mechanism in educating students at the College of Technology. Indeed, it was found that *returnable toys* are a most effective way to teach technology education.

PAPER BOOMERANG

The boomerang is made up of 3 sheets of cardboard (roughly 120 mm x 30 mm) by using a stapler, as shown in Figure 1. The side view of the boomerang is shown in Figure 1. The section of cardboard is folded slightly about a third of the way down to produce wing lift. The boomerang is bent roughly by 5 mm as a whole for a dihedral effect. The wing has a slight twist. The boomerang must be thrown lengthwise, not cross-wise, in a snapping action by the wrist, as shown in Figure 2.

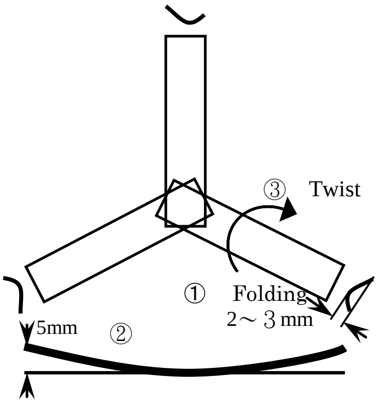

Figure 1: The taper boomerang.

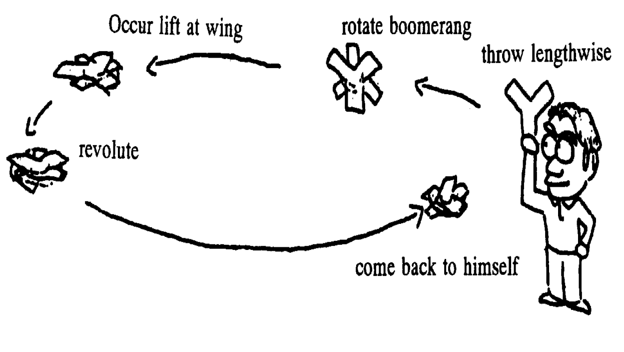

Figure 2: Throwing a paper boomerang.

The linear or straight velocity V_0 and angular velocity ω_0 when the boomerang is thrown lengthwise is shown in Figure 3. The moment around the *X*. axis occurs because of the difference in lift between the upper and lower part of the wing. Angular momentum *L* is $L = I_1\omega_0$, where I_1 is the moment of inertia around the x_3 axis. Variation *dL* of *L* during time *dt* is *Ndt*.

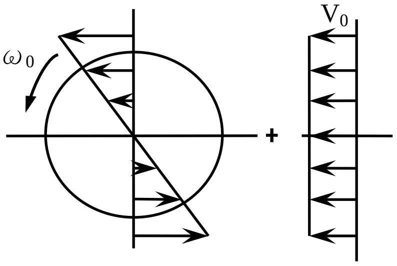

Figure3: Straight velocity (right) and angular velocity (left).

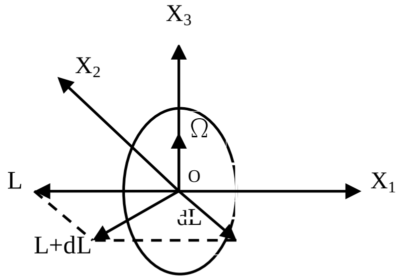

Figure 4: Coordinate axis.

Accordingly,

$$\frac{dL}{dt} = \Omega X L = N \tag{1}$$

$$\Omega = \frac{N}{I_1 \omega_0} \tag{2}$$

The direction of total lift F is perpendicular to X_1X_3.
The boomerang rotates in the X_1X_2 plane with period

$T = 2\pi / \Omega$.

When absolute axis is X Y

$$\frac{d^2X}{dt^2} = \frac{F}{m}\cos\Omega t \tag{3}$$

$$\frac{d^2Y}{dt^2} = \frac{F}{m}\sin\Omega t \tag{4}$$

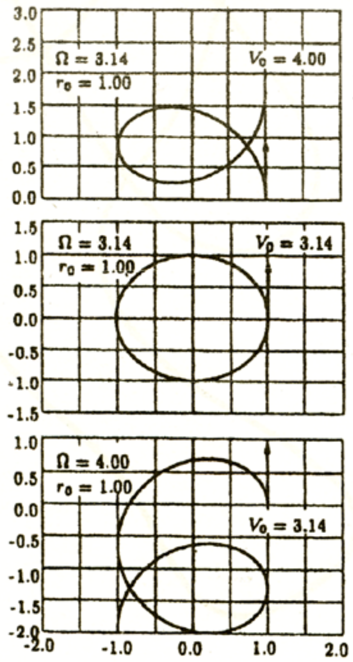

Figure 5: The loci of the boomerang.

When Equation (3) and Equation (4) are integrated with initial conditions, we find:

$$X = r_0 \cos\Omega t \tag{5}$$

$$Y = r_0 \sin\Omega t + (v_0 - r_0\Omega)t \tag{6}$$

The locus is the result of angular motion with radius *r*.0 and straight motion in the *Y* direction. Figure 5 shows these loci. The upper Figure shows a large straight velocity. The lower Figure shows a large angular motion. The boomerang returns to the thrower when the straight velocity coincides with the angular velocity.

BOOMERANG PLANE

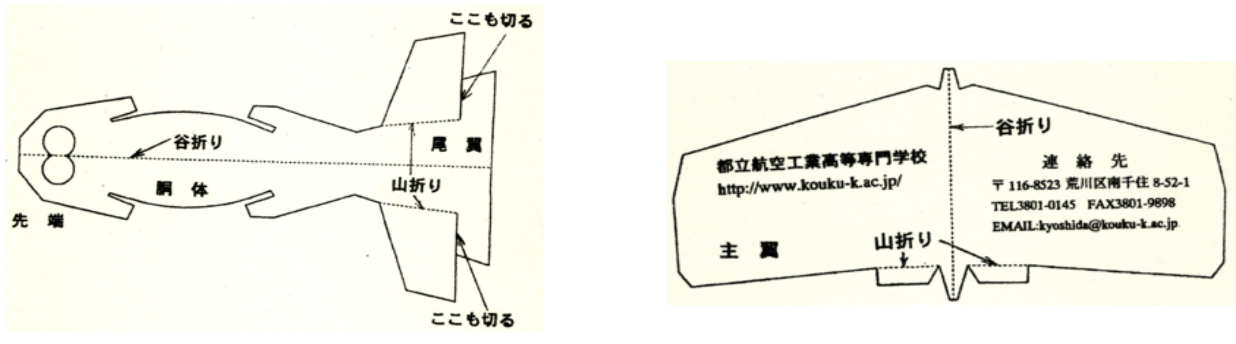

Figure 6: The boomerang plane.

Based on a Boomerang Plane sold in shops, the authors devised a Boomerang Plane model that is easy to clip out from a pattern. Figure 6 shows a simple model of the Boomerang Plane. The simple model is used for introducing technology to freshmen. The method used to manufacture and fly the Boomerang Stunt Plane is described as follows:

Cut the paper along the solid line. Make a *mountain fold* and *valley fold* along the dotted line (as per instructions provided). Attach the weight of one gram to the circle printed at on the nose of the fuselage. In Japan, a one-yen coin is used for this weight, because a one-yen coin weighs just one gram. Insert the wing along the bending chink of the fuselage. Fold down the tails. Catch hold of the centre of the fuselage and then the nose will face the ceiling. Stretch an arm outwards and throw the plane vertically upward. The plane will loop and return to you. The palm of the hand should face upward to catch the plane.

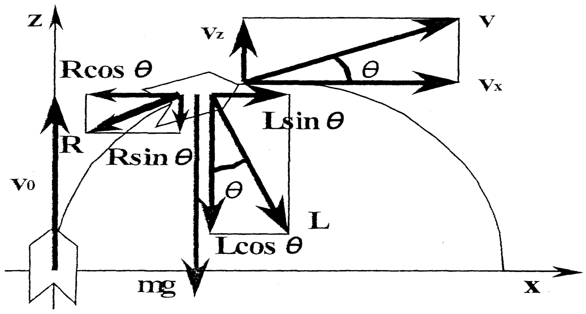

Figure 7: Coordinate axis.

Figure 7 shows the coordinate axis and the forces acting on the plane. Lift L is at right angles toward the flying direction, drag R is opposite to the flying direction and the force of gravity is mg. By adjusting the tail flap slightly, it is possible to change the attack angle of the wing. The lift coefficient C_L and drag coefficient C_D change when the attack angle is adjusted. The lift L can be expressed by Equation (7).

$$L = \frac{1}{2}\rho A C_L v^2 = \alpha v^2, \quad \alpha = \frac{\rho A C_L}{2} \tag{7}$$

Where ρ is the density of air, A is the area of the wings and the velocity of the plane. The plane is thrown upward with an initial velocity v_0 as shown in Figure 7. Drag is neglected because drag is much smaller than lift.

$$\frac{dx}{dt} = v_x = v\cos\theta, \quad \frac{dz}{dt} = v_z = v\sin\theta .$$

So, the elements of the x direction and the z direction are: $L_x = \alpha v^2 \sin\theta = \alpha v \frac{dz}{dt}$, $L_z = \alpha v^2 \cos\theta = \alpha v \frac{dx}{dt}$.

Equations of the plane's motion are expressed by the energy conservation law, as follows:

$$\frac{1}{2}mv^2 + mgz = \frac{1}{2}mv_0^2, \quad v = \sqrt{v_0^{\,2} - 2gz} \approx v_0 - \frac{g}{v_0}z,$$

$$\frac{d^2x}{dt^2} = \frac{\alpha}{m}\left(v_0 - \frac{g}{v_0}z\right)\frac{dz}{dt} \tag{8}$$

$$\frac{d^2z}{dt^2} = -g - \frac{\alpha}{m}\left(v_0 - \frac{g}{v_0}z\right)\frac{dx}{dt}. \tag{9}$$

Initial conditions at t=o are x=0, z=0, $\frac{dx}{dt} = 0$, $\frac{dz}{dt} = v_0$. By integrating Equation (8), Equation (9):

$$\frac{dx}{dt} = \frac{\alpha v_0}{m}z - \frac{\alpha g}{2mv_0}z^2 + C_1. \tag{10}$$

From the initial conditions, C_1=0. Then, $\frac{dx}{dt} = \frac{\alpha v_0}{m}z - \frac{\alpha g}{2mv_0}z^2$.

After substituting this equation for Equation (9), the following equation is obtained:

$$\frac{d^2z}{dt^2} = -g - \frac{\alpha^2}{m^2}\left(v_0^{\,2}z - \frac{3g}{2}z^2 + \frac{g^2}{2v_0^{\,2}}z^3\right).$$

The z^3 term is neglected because it is much smaller than the other terms,

$$\frac{d^2z}{dt^2}+n^2z+\beta z^2=-g\,, \tag{11}$$

where, $n^2=\frac{\alpha^2{v_0}^2}{m^2}$, $\beta=-\frac{3\alpha^2g}{2m^2}$

Equation (11) is a non-linear differential equation in z. To solve this equation, assume that $z=z_1+z_2+z_3\cdots$, where, z_1 is the first order term and z_2 is the second order term. After substituting for z_1 and z_2, the following equations are obtained.

$$\frac{d^2z_1}{dt^2}+n^2z_1=-g\,, \tag{12}$$

$$\frac{d^2z_2}{dt^2}+n^2z_2+\beta {z_1}^2=-g\,\cdot \tag{13}$$

The solution of Equation (12) from the initial conditions is as follows: $z_1=C_2\sin nt-\frac{g}{n^2}$.

After substituting this into Equation (13), the next equation is obtained:

$$\frac{d^2z_2}{dt^2}+n^2z_2=-g-\beta\left(C_2\sin nt-\frac{g}{n^2}\right)^2\cdot$$

The solution of this equation is:

$$z=z_1+z_2=C_3\sin nt-\frac{\beta C_2^2}{6n^2}\cos 2nt\;-\frac{\beta C_2 g}{n^3}t\cos nt-\frac{2g}{n^2}-\frac{\beta g^2}{n^6}-\frac{\beta C_2^2}{2n^2}\,. \tag{14}$$

$$\frac{dz}{dt}=C_3n\cos nt+\frac{\beta {C_2}^2}{3n}\sin 2nt\;-\frac{\beta C_2 g}{n^3}(\cos nt-nt\sin nt)\cdot \tag{15}$$

From the initial conditions, $C_2=\sqrt{-\frac{3g}{\beta}-\frac{3g^2}{2n^4}}$, $C_3=\frac{v_0}{n}+\frac{\beta g}{n^4}\sqrt{-\frac{3g}{\beta}-\frac{3g^2}{2n^4}}$.

The result of the integration of Equations (10) is:

$$x=\frac{\alpha v_0}{m}\int zdt-\frac{\alpha g}{2mv_0}\int z^2dt\,. \tag{16}$$

By Equation (14), x is obtained by numerical integration of Equation (16). The loci of the plane is obtained from these Equations, after substituting the following values: ρ=1.2kg/m^3, A=0.01m^2, v_0=6m/s, C_L=0.9, m=0.004kg.

Figure 8 is obtained by using Microsoft Excel. Figure 9 is the photograph at 0.1 sec intervals when throwing the plane upward. In comparing Figure 8 with Figure 9, the theoretical locus is shown to agree satisfactorily with the experimental one.

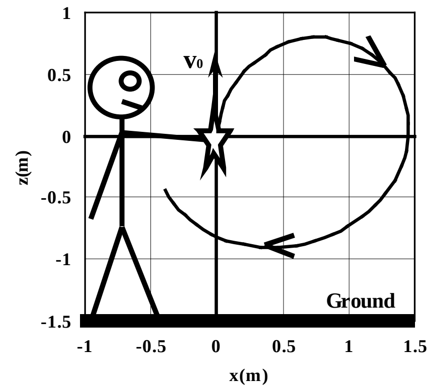

Figure 8: Theoretical locus.

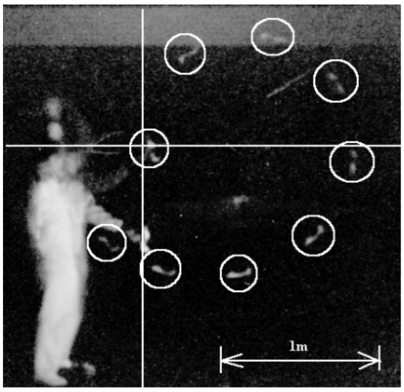

Figure 9: Real locus.

BAMBOOMERANG

A bamboomerang is a boomerang dragonfly made by bamboo in Japan, as shown in Figure 10. A bamboomerang has a slight weight at both ends and a spindle end. It is launched at a slight slant. Its flight is vertical, returning to the thrower's hands. This movement looks like the motion of a pendulum. The loci of bamboomerangs is analysed by simple equations of motion:

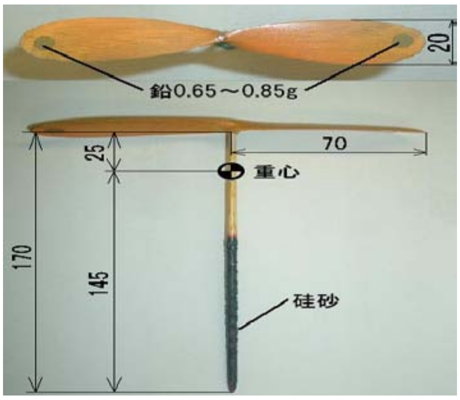

Figure 10: Bamboomerang.

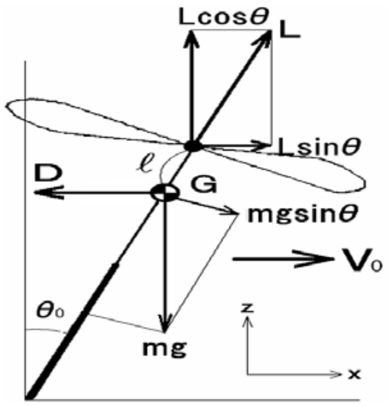

Figure 11: Coordinate axis.

Figure 11 shows the frame of reference for this analysis. Lift *L* acts on the direction of travel of the bamboomerang. Drag *D* acts in the opposite direction. Gravity *mg* acts perpendicularly downward from the centre of gravity. The bamboomerang flew with an initial angle θ_0 and initial velocity v_0. Equations of motion of the directions of x and z, and the angle θ become:

$$m\frac{d^2x}{dt^2} = L\sin\theta - D \tag{17}$$

$$m\frac{d^2z}{dt^2} = L\cos\theta - mg \tag{18}$$

$$J\frac{d^2\theta}{dt^2} = -mg\ell\sin\theta \tag{19}$$

where, m is the mass of the bamboomerang, J is the moment of inertia around the centre of gravity and l is the distance from the centre of the wing to the centre of gravity. Moreover, in this analysis, it was assumed there was a relation:

$$L = L_o e^{-\varepsilon t} \tag{20}$$

where, t is the flight time, L_0 is lift at the start of the flight and ε is a constant.

Firstly, if θ is assumed to be small, $\sin\theta \fallingdotseq \theta$. Therefore, Equation (19) becomes:

$$\frac{d^2\theta}{dt^2} + {\omega_n}^2\theta = 0 \tag{21}$$

Where, $\omega_n = \sqrt{\frac{mg\ell}{J}}$. When Equation (20) is solved under the initial conditions $t = 0, \theta = \theta_0, \dot{\theta} = 0$, it becomes:

$$\theta = \theta_0 \cos\omega_n t \tag{22}$$

Secondly, to solve Equation (17); θ is assumed to be small, and using Equations (19), (20) and (21), Equation (17) then becomes:

$$\frac{d^2x}{dt^2} = \frac{L_0\theta_0}{m}e^{-\varepsilon t}\cos\omega_n t - \frac{D}{m} \tag{23}$$

The general solution x_n and the particular solution x_s of the homogeneous Equation (23) is:

$$x_n = \frac{C_1}{m}t + \frac{C_2}{m} \tag{24}$$

$$x_s = Ae^{-\varepsilon t}\cos\omega_n t + Be^{-\varepsilon t}\sin\omega_n t + Ct^2 \tag{25}$$

where, C_1, C_2, A, B and C are constant. When Equation (25) is differentiated twice, and substituted for Equation (23), the following equation is obtained:

$$A = \frac{\alpha L_0\theta_0}{m(\alpha^2+\beta^2)},\quad B = -\frac{\beta L_0\theta_0}{m(\alpha^2+\beta^2)},\quad C = -\frac{D}{2m}$$

where, $\alpha = \varepsilon^2 - \omega_n^2,\quad \beta = 2\varepsilon\omega_n$. Therefore, when it is assumed $\dfrac{L_0\theta_0}{m(\alpha^2+\beta^2)} = \gamma$, because x_s is:

$$x = \alpha\gamma e^{-\varepsilon t}\cos\omega_n t - \beta\gamma e^{-\varepsilon t}\sin\omega_n t - \frac{D}{2m}t^2 \tag{26}$$

the entire general solution is:

$$x = \frac{C_1}{m}t + \frac{C_2}{m} + \alpha\gamma e^{-\varepsilon t}\cos\omega_n t - \beta\gamma e^{-\varepsilon t}\sin\omega_n t - \frac{D}{2m}t^2 \tag{27}$$

When Equation (27) is differentiated, and initial condition substituted, t=0，x=0， $\dot{x} = v_0$, the final general solution is:

$$x = \{v_0 + (\varepsilon\alpha + \omega_n\beta)\gamma\}t + \gamma(\alpha\cos\omega_n t - \beta\sin\omega_n t)e^{-\varepsilon t} - \frac{D}{2m}t^2 - \alpha\gamma \tag{28}$$

Finally, to solve Equation (18), when θ is assumed to be small and using Equation (19) and Equation (20), Equation (18) becomes:

$$\frac{d^2z}{dt^2} = \frac{L_0}{m}e^{-\varepsilon t} - g \tag{29}$$

The general solution z_n and the particular solution z_s of the homogeneous Equation (29) is:

$$z_n = \frac{C_3}{m}t + \frac{C_4}{m} \tag{30}$$

$$z_s = Ee^{-\varepsilon t} + Ft^2 \tag{31}$$

where, C_3, C_4, E and F are constants. When Equation (31) is differentiated twice, and substituted into Equation (29), it becomes:

$$E = \frac{L_0}{m\varepsilon^2},\quad F = -\frac{g}{2}$$

Therefore, the entire general solution is:

$$z = \frac{C_3}{m}t + \frac{C_4}{m} + \frac{L_0}{m\varepsilon^2}e^{-\varepsilon t} - \frac{g}{2}t^2 \tag{32}$$

When Equation (32) is differentiated, and initial condition substituted, t=0，z=0， $\dot{z} = 0$, the final general solution is:

$$z = \frac{L_0}{m\varepsilon}t + \frac{L_0}{m\varepsilon^2}(e^{-\varepsilon t} - 1) - \frac{g}{2}t^2 \tag{33}$$

Lift L and drag D are shown by the next equations.

$$L = 1/2C_L\rho V^2S,\ D = 1/2C_D\rho V^2S \tag{34}$$

When C_L=1.2，C_D=0.17, ρ=1.2kg/m^3，V=9m/s，S=0.0014m^3 are substituted into Equation (34), L=0.08N, D=0.0116N are obtained. When m=0.007kg，v_0=3m/s，θ_0=10°，L=0.08N，D=0.0116N, ω_n=9rad/s are substituted into theoretical equations, then Figure 12 is obtained. Figure 13 shows the real locus. Hence, it is found that the theoretical locus agrees well with the real locus.

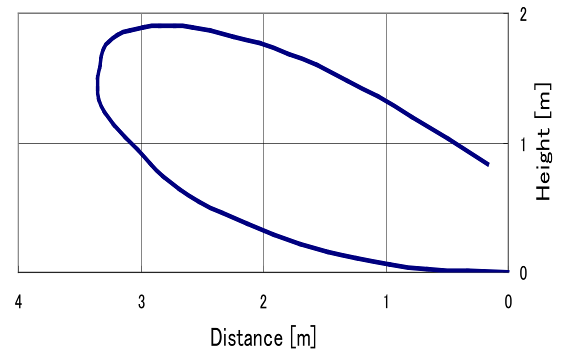

Figure 12: The theoretical locus.

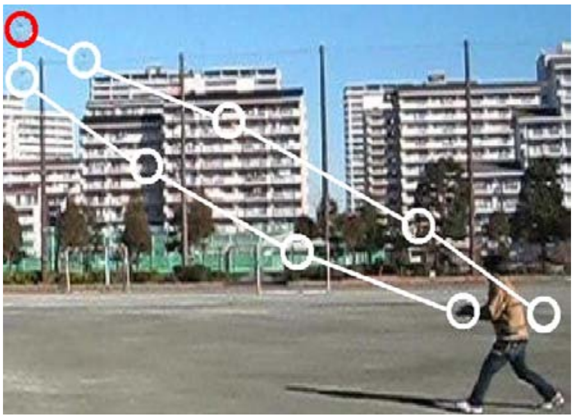

Figure 13: The real locus.

PAPER CUP BOOMERANG

The authors received a boomerang paper cup from the Nagoya Science Museum. Two paper cups are connected by Scotch tape, as shown in Figure 14. Three rubber rings are fastened to it. The connected paper cup is tugged upward by a rubber ring, as shown in Figure 14. After the paper cup is released, the paper cup flies upward suddenly and returns to the person throwing it.

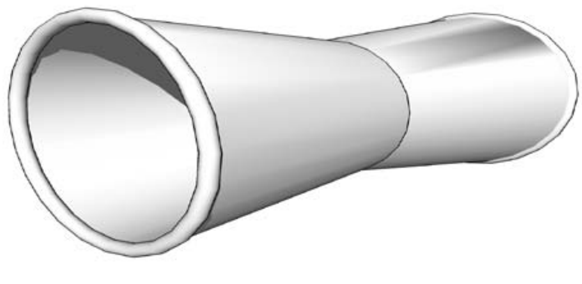

Figure 14: Paper cup boomerang.

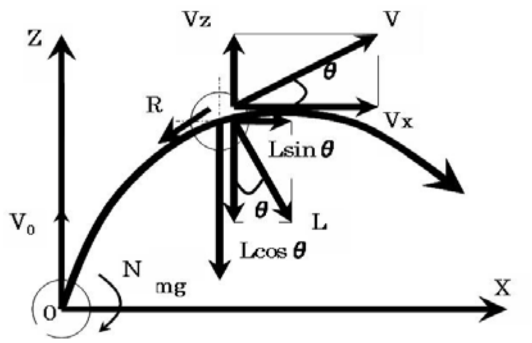

Figure 15: Coordinate axis.

The equation of motion is the same one as the boomerang plane. When the paper cup is thrown with the initial velocity V_0 and initial rotation N as shown in Figure 15, Lift L, Drag R and Gravity mg act on the paper cup. R is neglected because it is a slight force. The Lift L is proportion to velocity V by the *Magnus effect*. $L = \rho V \Gamma h = \alpha V$

where, ρ is air density. The circulation around the cup is $\Gamma = \pi d q_\theta$，the rotational velocity is $q_\theta = \pi d N$，the length of the cup is h. The equations of motion are as follows:

$$m\frac{d^2x}{dt^2} = L\sin\theta = \alpha V \sin\theta = \alpha\frac{dz}{dt} \tag{35}$$

$$m\frac{d^2z}{dt^2} = -mg - L\sin\theta = -mg - \alpha\frac{dx}{dt} \tag{36}$$

For the initial conditions $t = 0$, $x = z = \frac{dx}{dt} = 0$，$\frac{dz}{dt} = V_0$ $\frac{dz}{dt} = V_0$ Equation (35) and Equation (36) are integrated, giving:

$$z = \left(\frac{m}{\alpha}\right)^2 g\cos\frac{\alpha}{m}t - \frac{m}{\alpha}V_0\sin\frac{\alpha}{m}t - \left(\frac{m}{\alpha}\right)^2 g \tag{37}$$

$$x = \left(\frac{m}{\alpha}\right)^2 g \sin\frac{\alpha}{m}t - \frac{m}{\alpha}V_0 \cos\frac{\alpha}{m}t - \frac{m}{\alpha}gt + \frac{m}{\alpha}V_0 \tag{38}$$

The theoretical locus is shown in Figure 16 and the real locus is shown in Figure 17. It can be seen that the theoretical locus coincides very well with the real locus.

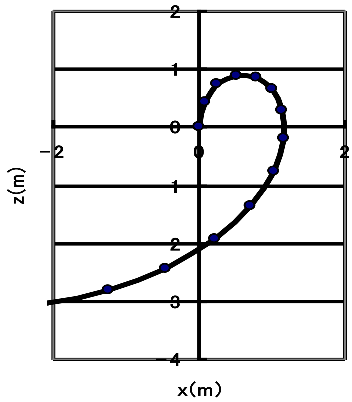

Figure 16: The theoretical locus.

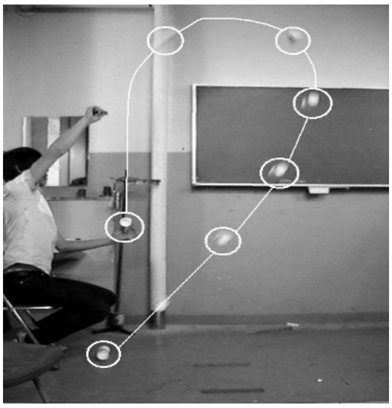

Figure 17: The real locus.

CONCLUSIONS

Several returnable flying objects were analysed in this paper. These objects included the paper boomerang, the boomerang and the paper cup boomerang. The loci of the bamboomerangs were analysed using simple equations of motion. It has been found that the theoretical results coincided closely with the experimental results.

In their effort to introduce new technologies to schoolchildren, the authors taught them how to use the bamboomerangs. Analyses of the loci of such flying objects, were introduced in this paper. To prove the theoretical aspects covered in this paper, it is envisaged that the authors will demonstrate the various types of flying object, including the bamboomerangs, while presenting their paper at the *1st World Conference on Technology and Engineering Education* in Kraków, Poland.

Based on the practical experiments and the responses by the schoolchildren, it can be concluded that the bamboomerangs are good teaching materials for educating them about technology.