

The use of numerical methods in teaching selected topics in circuit theory based on MATLAB

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ABSTRACT: This paper addresses the challenge of developing techniques for the effective teaching of electric circuit theory. It describes the examples and experience of using mathematical programs to solve tasks in the computational theory of electrical circuits at the AGH University of Science and Technology in Kraków and the State Higher Vocational School in Tarnów, Poland. This paper presents two examples of the use of numerical methods. The first example shows the calculation of a complex three-phase circuit. The second example presents an analysis of an electrical circuit containing an element with nonlinear current-voltage characteristics, such as a tunnel diode. These examples concentrate on the numerical capabilities of the computer program MATLAB for solving linear and nonlinear equations and its plotting capabilities. Through these abilities, more time can be devoted to the analysis of phenomena that take place, while electric circuits are being solved. Therefore, the authors perceive mathematical programs as a very useful didactical tool in teaching electric circuit theory.

INTRODUCTION

Electric circuit theory plays an important role in the understanding of electric and magnetic phenomena, and it allows students to understand the principles of electrical devices. Many branches of electrical engineering, such as electronics, power, electric machines, communications, control and instrumentation are based on electric circuit theory [1][2]. Therefore, a fundamental electric circuit theory course is most important for electrical engineering students in the first stage of their education, as well as students specialising in other disciplines of the physical and technical sciences. This is because circuits are an excellent model for the study of energy and power systems in general, and also because of the applied mathematics, physics and topology involved.

The most typical forms for teaching circuit theory at university are the following:

- Lectures;
- Auditorium exercises - classes;
- Laboratory exercises;
- Homework - projects.

The *theory of circuits* curriculum always includes a few essential topics. They are briefly presented below:

- Current, voltage, electrical power, basic components and electric circuits topology;
- Kirchhoff's Current and Voltage Laws and Ohm's Law;
- Resistive Network Analysis:
    - Classical method;
    - The mesh current method;
    - The nodal voltage method.
- Supporting theorems and rules:
    - The Principle of Superposition;
    - Source and resistance transformation as a means of simplifying circuits;
    - Thévenin's and Norton's Theorems of equivalent circuits;
    - Maximum power transfer;
    - Numerical and graphical (load-line) analysis of nonlinear circuit elements;

- $\Delta \leftrightarrow Y$ transformations.

- Sinusoidal steady-state analysis – AC Network analysis;
- Magnetically coupled circuits;
- AC circuit power analysis;
- Three-phase circuits;
- Complex waveforms and Fourier analysis;
- Transient analysis:
  - Natural response of electric circuits;
  - Transients and Laplace transforms;
  - Differential equations for circuits and analysis by state variables.
- Two-port networks;
- Filter networks;
- Nonlinear circuits;
- Elements of circuit synthesis.

Practical classes with students are conducted for enhancing and supplementing knowledge obtained during the lectures. Exercises are aimed at teaching students how to solve tasks and problems of electric circuits. Exercises are associated with solving computational tasks of circuit theory. Generally, there are three stages of this process:

- Formulation of the circuit equations depending on the chosen method of analysis;
- Solution of the circuit equations;
- Discussion of results.

The first stage requires the knowledge of theory and the methods of circuit analysis. The time for its realisation depends on the complexity of the problem being solved. The final stage enables students to understand the phenomena occurring in electric circuits and the way electrical systems operate. This is possible due to to the analysis and discussion of the results. The second stage is very time-consuming. It takes over 80% of the total time spent on the task. During mathematical computations, students very often make mistakes. This leads to incorrect results and makes the proper execution of the third stage impossible. Therefore, the application of computers in the second stage reduces the amount of time spent by students to perform computational tasks and eliminates calculation errors. Nowadays, computers allow for relatively quick calculations and the elimination of calculation errors [4][5].

There are many mathematical programs that allow for symbolic and numerical calculations. Some of them are free software (e.g. Maxima, Scilab) or commercial programs, such as MATLAB, Mathcad, Mathematica and Maple. Each of these programs has a powerful mathematical programming environment, with great possibilities for numeric and symbolic calculations, as well as computer simulations. All of them can be used to support the teaching of electrical engineering and many other disciplines. This paper describes the use of MATLAB in supporting the analysis of selected electrical circuits. The interactive programming and versatile graphics of MATLAB is notably effective in exploring some of the characteristics of devices and electrical circuits.

The MATLAB package enables users to solve many advanced numerical problems rapidly and efficiently; MATLAB is used to teach field theory, control systems circuit theory, communication theory, filter design and many other areas of specialty. Matrix functions of MATLAB are versatile when analysing data obtained from technical experiments, and the graphical features of MATLAB are especially useful for the display of the frequency response of systems and illustrating the principles and concepts of semiconductor physics. The interactive programming and versatile graphics of MATLAB are notably effective in exploring some of the characteristics of devices and electrical circuits [6].

An earlier paper presented examples of using the mathematical programmes to solve tasks in computational theory of electrical circuits at the AGH University of Science and Technology in Kraków and the State Higher Vocational School in Tarnów, Poland [3]. In this paper, two examples of the use of numerical methods are presented, which are typical problems that could be solved as illustrative examples assigned as tasks in an exercise set as homework. The first example shows the calculation of a complex three-phase circuit. The second example presents an analysis of an electrical circuit containing an element with nonlinear current-voltage characteristics, such as a tunnel diode.

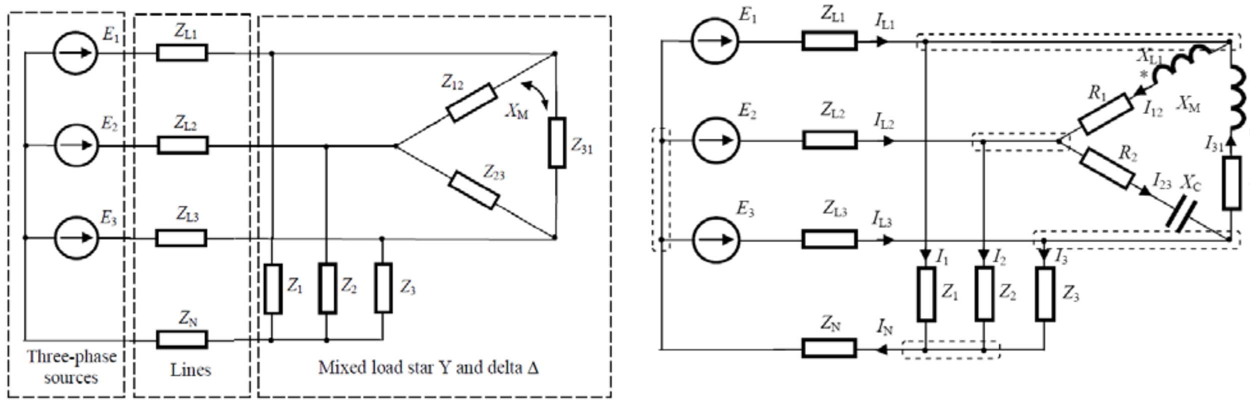
THREE-PHASE SYSTEMS

Generation, transmission and distribution of electricity is accomplished by a three-phase alternating current system. A three-phase AC supply is carried by three conductors called lines. The currents in these conductors are known as line currents I_L. Sources of three-phase supplies, i.e. generators, are usually connected as a star, whereas three-phase transformer windings, motors and other loads may be connected as either a star or delta. Loads of three-phase systems may be balanced or unbalanced [2].

The first example is aimed at making students familiar with the basic knowledge of three-phase systems necessary for further research. The concepts of AC steady-state analysis are applied to analyse the circuit; concepts that are introduced during an earlier part of the circuit theory course. The analysis of circuits containing such elements as a resistor *R*, inductor *L*, capacitor *C* and mutual inductance *M*, excited by sinusoidal three-phase sources, is considerably simplified when phasor voltages *U* and phasor currents *I* are used.

The analysis of a three-phase network involves determining each of the unknown branch currents and voltages. Consider a mixed delta and star connected unbalanced load, as shown in Figure 1. In this circuit the balanced three-phase generator voltage E = 230 volts. Line impedances and the load of the star's impedances are respectively: $Z_{L1} = Z_{L2} = Z_{L3} = Z_N = 0.1 + j0.1\ \Omega$, $Z_1 = 10 + j10\ \Omega$, $Z_2 = 10 + j10\ \Omega$, $Z_3 = 10 + j10\ \Omega$.

Delta load impedances are, respectively: $Z_{12} = R_1 + jX_{L1}$, $Z_{23} = R_2 - jX_C$, $Z_{31} = R_3 + jX_{L3}$, where: $R_1 = 5\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 5\ \Omega$ and $X_{L1} = 5\ \Omega$, $X_C = 5\ \Omega$, $X_{L3} = 5\ \Omega$, $X_M = 5\ \Omega$. Now, find all the line currents and load currents, and draw a complete phasor diagram for the line currents, delta currents and star currents.



a) b)

Figure 1: The three-phase circuit; a) Y-connected three-phase generator with mixed Delta and Star connected unbalanced load; b) Circuit diagram with cut-set, currents in lines I_L and currents in load I_Y, I_Δ.

As shown in Figure 1a, for all the impedances in the circuit, a current variable with arbitrary polarity is assigned. The analysed circuit has *N* = 5 cut-set (super node), where *N* is the total number of the cut-set in the circuit. Now, write Kirchhoff's Current Law equations for an *N*-1 cut-set. The algebraic sum of currents at the super node is zero.

$$\begin{aligned} & I_{L1} - I_1 - I_{12} + I_{31} = 0 \\ & I_{L2} - I_2 + I_{12} - I_{23} = 0 \\ & I_{L3} - I_3 + I_{23} - I_{31} = 0 \\ & I_1 + I_2 + I_3 - I_N = 0 \end{aligned} \quad (1)$$

The relationship between the current phasor *I* and the voltage phasor *V* on the branch impedance Z is as follows:

$$V = Z \cdot I \quad (2)$$

Expression (2) is referred to as Ohm's Law for AC circuits.

Express the impedance of each branch as the sum of real and imaginary components, as follows:

$$Z_{12} = R_1 + jX_{L1} \qquad Z_{23} = R_2 - jX_C \qquad Z_{31} = R_3 + jX_{L3} \quad (3)$$

Applying Kirchhoff's voltage equations for *B-N+1* = 6 independent loops, where *B* is the number of branches in the circuit, results in:

$$\begin{aligned} & E_1 - Z_{L1} \cdot I_{L1} - (R_1 + jX_{L1}) \cdot I_{12} - jX_M \cdot I_{31} + Z_{L2} \cdot I_{L2} - E_2 = 0 \\ & E_2 - Z_{L2} \cdot I_{L2} - (R_2 - jX_C) \cdot I_{23} + Z_{L3} \cdot I_{L3} - E_3 = 0 \\ & E_1 - Z_{L1} \cdot I_{L1} + (R_3 + jX_{L3}) \cdot I_{31} - jX_M \cdot I_{12} + Z_{L3} \cdot I_{L3} - E_3 = 0 \\ & E_1 - Z_{L1} \cdot I_{L1} - Z_1 \cdot I_1 - Z_N \cdot I_N = 0 \\ & E_2 - Z_{L2} \cdot I_{L2} - Z_2 \cdot I_2 - Z_N \cdot I_N = 0 \\ & E_3 - Z_{L3} \cdot I_{L3} - Z_3 \cdot I_3 - Z_N \cdot I_N = 0 \end{aligned} \quad (4)$$

There is a sufficient number of simultaneous linear equations to solve for unknown currents at branches I_B. The above set of equations (4) in matrix notation takes the following form:

$$\text{Matrix} \cdot I_B = \text{Right\_Side} \tag{5}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ Z_{L1} & -Z_{L2} & 0 & 0 & 0 & 0 & 0 & Z_{12} & 0 & jX_M \\ 0 & Z_{L2} & -Z_{L3} & 0 & 0 & 0 & 0 & 0 & Z_{23} & 0 \\ -Z_{L1} & 0 & Z_{L3} & 0 & 0 & 0 & 0 & -jX_M & 0 & Z_{31} \\ Z_{L1} & 0 & 0 & Z_N & Z_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{L2} & 0 & Z_N & 0 & Z_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{L3} & Z_N & 0 & 0 & Z_3 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{L1} \\ I_{L2} \\ I_{L3} \\ I_N \\ I_1 \\ I_2 \\ I_3 \\ I_{12} \\ I_{23} \\ I_{31} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E_1 - E_2 \\ E_2 - E_3 \\ E_3 - E_1 \\ E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{6}$$

To solve the set of equations (6) and find B = 10 currents the program MATLAB was used. The following is the MATLAB script for the solution:

```
E=230;E1=E;E2=E*exp(-j*2/3*pi);E3=E*exp(j*2/3*pi); ET=[E1 E2 E3]';
ZL1=0.2+j*0.1;ZL2=0.2+j*0.1;ZL3=0.2+j*0.1;ZN=0.3+j*0.1
Z1=7+j*4;Z2=5-j*5;Z3=5-j*5;
R1=10;R2=15;R3=5;XL1=8;XL3=10;Xc=5;XM=3;Z12=R1+j*XL1;Z23=R2-j*Xc;Z31=R3+j*XL3;
Matrix=[1 0 0 0 -1 0 0 -1 0 1
    0 1 0 0 0 -1 0 1 -1 0
    0 0 1 0 0 0 -1 0 1 -1
    0 0 0 -1 1 1 1 0 0 0
    ZL1 -ZL2 0 0 0 0 0 Z12 0 -j*XM
    0 ZL2 -ZL3 0 0 0 0 0 Z23 0
    -ZL1 0 ZL3 0 0 0 0 -j*XM 0 Z31
    ZL1 0 0 ZN Z1 0 0 0 0 0
    0 ZL2 0 ZN 0 Z2 0 0 0 0
    0 0 ZL3 ZN 0 0 Z3 0 0 0]
Right_Side=[0 0 0 0 E1-E2 E2-E3 E3-E1 E1 E2 E3]'
I=inv(Matrix)*Right_Side;
IL1=I(1); IL2=I(2); IL3=I(3);IN=I(4); IL=[IL1 IL2 IL3]'
I1=I(5);I2=I(6);I3=I(7);                Iy=[I1 I2 I3]'
I12=I(8); I23=I(9); I31=I(10);          ID=[I12 I23 I31]'
subplot(1,3,1),compass(IL),hold on,compass(IN,'k'),title('Currents I_L and I_N')
subplot(1,3,2),compass([Iy;IN]),compass(IN,'k'),title('Currents I_Y and I_N')
subplot(1,3,3),compass(ID),title('Currents I_D')
```

The results of a MATLAB script are shown below, in Figure 2 and Table 1.

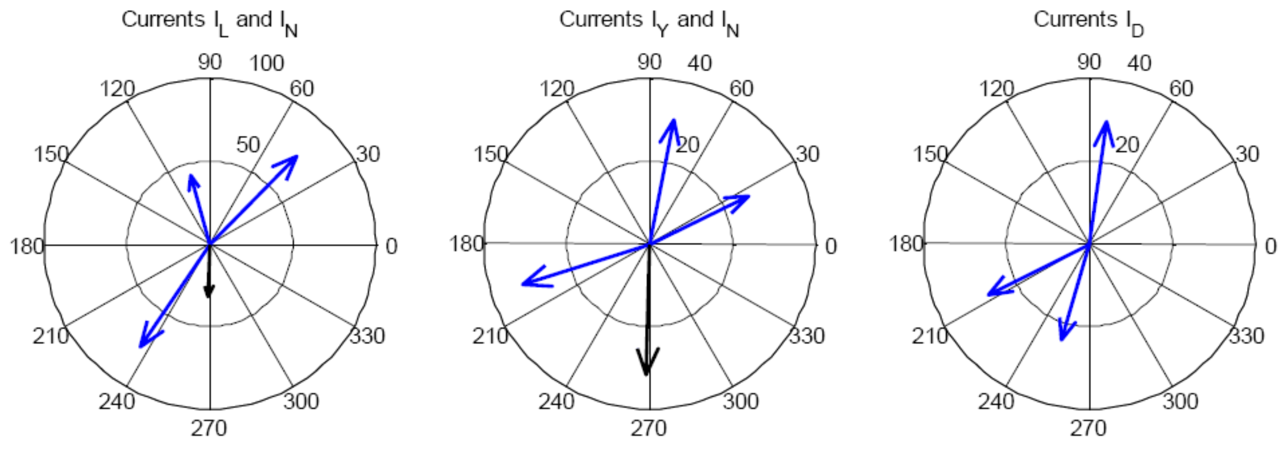


Figure 2: A phasor diagram for line currents I_L, star currents I_Y and delta currents I_Δ.

Table 1: Values of line, star and delta currents.

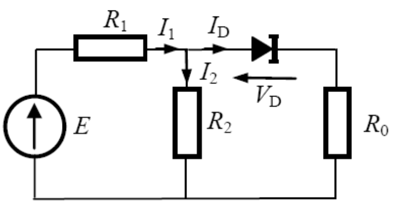
Currents of line I_L	Currents of stars I_Y	Currents of delta I_Δ
$\mathbf{I}_L$ = 52.3218 +52.8851i -41.3931 -62.1979i -11.6005 +41.0082i I_N = -0.6718 -31.6955i	$\mathbf{I}_Y$ = 23.6713 +11.3950i -30.3931 - 9.6992i 6.0500 +29.9997i	$\mathbf{I}_\Delta$ = 4.3626 +29.3305i -6.6374 -23.1682i -24.2880 -12.1597i

NONLINEAR CIRCUIT WITH TUNNEL DIODE

A nonlinear circuit consists of at least one nonlinear element, not counting the independent voltage and current sources. As opposed to linear circuits, which consist of linear elements only, nonlinear circuits may possess multiple solutions or may not possess any solution at all. A circuit element is called nonlinear, if its constitutive relationship between the voltage and the current is a nonlinear function or a nonlinear relation. Analysis of nonlinear circuits is more difficult than the analysis of linear circuits. Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) express linear relationships between a circuit's voltages or currents [7]. The nonlinear relations between circuit variables stem from the element relations. As a result, the equations governing the behaviour of a nonlinear circuit are nonlinear.

Consider the nonlinear circuit shown in Figure 3a. The circuit consists of a DC voltage source E, resistances R_1, R_2, R_3 and a tunnel diode. The aim is to determine the diode current I_D and the diode voltage V_D.

a)



b)

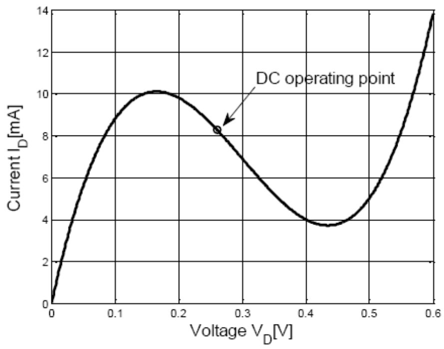


Figure 3: a) Nonlinear circuit with a tunnel diode; b) The approximated *i-v* characteristic of the tunnel diode.

Applying KCL for the node, and writing KVL around the loop, gives the following set of equations:

$$\begin{aligned} & I_1 - I_2 - I_D = 0 \\ & E - R_1 I_1 - R_2 I_2 = 0 \\ & R_2 I_2 - V_D - R_2 I_D = 0 \end{aligned} \tag{7}$$

The diode current and voltage are related by the diode equation.

$$I_D = c_1 V_D^3 + c_2 V_D^2 + c_3 V_D \tag{8}$$

The program MATLAB was used to solve the set of Equations (7) with the characteristic (8) to find the diode current I_D and the diode voltage V_D. The following MATLAB script was used to produce a solution:

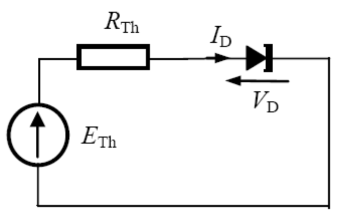
```
function Eq=dioda_t(x)
global R1 R2 R3 C1 C2 C3 E1
%I1=x(1),I2=x(2),ID=x(3),VD=x(4)
Eq=[x(1)-x(2)-x(3)
    E1-R1*x(1)*10^-3-R2*x(2)*10^-3
    x(4)-R2*x(2)*10^-3+R3*x(3)*10^-3
    x(3)-C1*x(4)^3-C2*x(4)^2-C3*x(4)];

% Nonlinear circuit
global R1 R2 R3 C1 C2 C3 E1
R1=300;R2=150;R3=50;C1=650; C2=-585; C3=140;E1=4.5;
% charakteristic v-i of tunnel diode %ID=C1*VD^3+c2*VD^2+C3*VD
C=[C1 C2 C3 0];VD=0:0.001:0.6;ID=polyval(C, VD);
plot(VD,ID,'LineWidth',2.5,'color','k'),grid on, hold on,axis([0 0.6 0 14])
```

```
xlabel('Voltage V_D[V]'),ylabel('Current I_D[mA]')
%calculation of the nonlinear circuit
x0=[1 1 8 0.3]; x = fsolve(@dioda_t,x0);sol=x'
plot(x(4),x(3),'ok','LineWidth',2)
result=sprintf('I1=%0.3f mA, I2=%0.3f mA, ID=%0.3f mA, VD=%0.3f V',x)
```

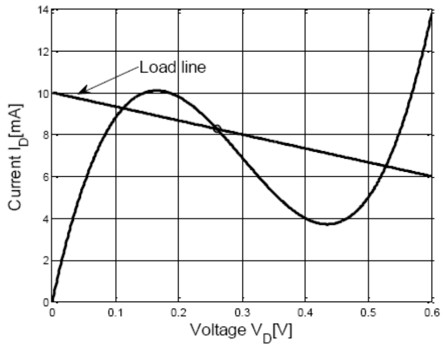
The results of the calculation are presented below:

$I_1 = 12.755$ mA, $I_2 = 4.491$ mA, $I_D = 8.264$ mA, $V_D = 0.260$ V.

An alternative graphical solution uses Thévenin's Theorem. This theorem involves replacing what may be a complicated network of sources and linear impedances with a simple equivalent circuit. Thévenin's equivalent circuit is shown in Figure 4a, and appropriate characteristics can be seen in Figure 4b.



a)



b)

Figure 4: a) Thévenin's equivalent circuit; b) Load line and diode forward characteristic.

Thévenin's equivalent circuit has a voltage source E_{Th} and the resistance R_{Th} given by the formulas below:

$$E_{Th} = \frac{R_2}{R_1 + R_2} E$$
$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} + R_3 \qquad (9)$$

Thévenin's equivalent circuit's load line is given by the line equation:

$$I = -\frac{1}{R_{Th}} V + \frac{E_{Th}}{R_{Th}} \qquad (10)$$

The circuit's load line is obtained by applying Kirchhoff's Voltage Law. The intersection of the diode's polynomial characteristic and the load line provides the circuit's DC operating point.

The following continued MATLAB script creates a graphical solution Figure 4 b).

```
%The characteristic i-u Thévenin,s source
RT=R1*R2/(R1+R2)+R3; ET=R2/(R1+R2)*E1;
I= ET*10^3/RT -VD*10^3/RT;
plot(VD,I,'LineWidth',2.5)
```

CONCLUSIONS

In this paper, the authors present how the mathematic software program MATLAB can be used effectively in the analysis of electrical circuits. Without performing many time-consuming computations, students are able to obtain fast results. The MATLAB code demonstrated in the paper is relatively simple. It is easy to draw and animate the solution too. In the analysis of nonlinear circuits, students use the numerical solution. This helps the students to understand that some problems can only be solved by numerical methods. The students learn how complex practical problems can be, and also appreciate the need of computer solving.

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