

Mathematics in engineering education

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ABSTRACT: Mathematics is a crucial language in all engineering courses and research in which mathematical modelling, manipulation and simulation are used. It is widely recognised, though, that engineering mathematics courses are regarded by students as a very difficult part of the engineering education curricula. This is reflected in engineering students' performance at the end of each semester when examinations for these courses are conducted. The main objective of this paper is to overview some concepts, ideas and questions with regard to mathematics as a fundamental subject of engineering studies.

INTRODUCTION

Undoubtedly, an engineer should have a good command of fundamental mathematics. The objective of teaching mathematics to engineering students is to find the right balance between practical applications of mathematical equations and in depth understanding of living situations.

Mathematics is a crucial language in all engineering courses and research in which mathematical modelling, manipulation and simulation are used extensively. Therefore, engineering mathematics courses are regarded as difficult courses in engineering education curricula. This is reflected in engineering students' performance at the end of each semester in these courses.

At all levels of education, the traditional method of teaching is lecture-based. The description of a lecture is conducted by using this traditional method. Even though this method is able to provide students with knowledge and produce graduates, the attention of the majority of students is superficial and their focus is on the development of exam-passing abilities.

In teaching engineering mathematics courses to engineering students, it has subsequently been found that they encounter difficulties and act indifferently toward this learning method. Students often regarded engineering mathematics courses as uninteresting and a difficult part of the engineering curriculum. The main problems are first year students' varying levels of knowledge in mathematics and low participation in teaching. Other possible factors are motivation, ability to work independently and acclimatisation to the university study environment.

On the other hand, the impact of mathematical thinking skills on engineers will enable them to use mathematics in their practice [1]. The attitude of engineering undergraduates toward mathematics was studied by Miika et al and this provides a better understanding of engineering students' actual knowledge and the lack of knowledge in mathematics [2].

It addresses long term needs for abilities and skills because these impose strong constraints on secondary and even on elementary education. The goal of higher education is to produce competent engineers. To do this, the university's academic staff need students to bring certain skills with them from high school and develop them further.

Mathematics is a fundamental subject for all engineering courses and research in engineering, but current educational approaches do not provide enough functionality to bring it to life. Current introductions to problems have passive understanding as their goal, rather than getting students to do things with them. A more active approach could be successful in real life settings. An overview of the structure and practice of mathematics was given by Quinn in 2011 [3].

Precision and Functionality

Mathematics has elaborated a set of explicit rules of reasoning. People often have trouble learning to use these rules effectively, because rules in most other systems are sufficiently unclear and ineffective, and precision is a waste of time, whereas in mathematics anything less than full precision is a waste of time.

Students should be taught to record their reasoning in a way that can be checked for errors. Criteria for good work formats are:

- Record enough detail so reasoning can be constructed and checked for errors;
- Be compact and straightforward;
- Help to organise the work in ways consistent with human cognitive constraints.

The most important strategy is the teacher's diagnosis of errors that students cannot find themselves. Wrong answers should be corrected. The student should explain his or her reasoning following his or her written work record. If the record is incorrect, steps were skipped or appropriate templates were not used, then, the work should be redone. If the work record is appropriate, then, it can be reviewed and mistakes quickly corrected.

Definitions and Key Statements

The important objects, terminology and properties should be given by brief and precise formulations:

- The formulations should be constructed primarily by professional mathematicians with education feedback.
- Students should memorise them, so they can be reproduced exactly. Definitions provide key points and functional understanding deepens around the definitions.
- Explanation of what a definition means is best given after the definition, not before. Putting too much explanation first almost guarantees confusion.

Intuition

A great deal of mathematical activity is guided by intuition. Intuition developed by working with a good definition is often effective. Intuition based on the meaning of a physical analogue is almost never satisfactory, but is acceptable if passive understanding is the goal.

In practice, it is not enough just to have good basic mathematics knowledge. An engineer is also required to have good generic skills, such as good communication skills, positive thinking and to be able to work independently. A mathematical model is developed to represent a physical phenomenon and this model is analysed and solved mathematically. A mathematical conclusion about the model is reliable when the connection between the physical situation and the model is appropriate.

Mathematics is a basic subject for students of engineering. In structuring the topics, the goal is to teach useful methods to future engineers. For future studies, the methods and simple applications are essential. Considering the main objective of mathematical training at technical universities is to teach students to solve applied problems. Mathematical skills are used to describe and cope with a wide range of problems. These key skills are about:

- understanding when particular techniques should be used;
- how to carry them out accurately; and
- which techniques should be applied in particular situations.

THE APPLICATION OF DIFFERENTIAL EQUATIONS

Many natural laws outline a relationship between the rates of change of various quantities, rather than the relationship between the quantities themselves.

The rate of change of a quantity is represented by the so-called derivative of this quantity. An equation involving derivatives is called a differential equation.

Differential equations have had a great influence on the history of science and demonstrate that mathematical methods can be applied to the real world. Some of the uses of differential equations are shown below, starting with the origin and, then, showing later applications in other fields of science [4][5].

The subject of differential equations has its primary historical origin in Newtonian mechanics, so it is necessary to begin by saying something about Newton and his results. Isaac Newton was born on Christmas Day 1642. It was a premature birth and according to the tradition, he was *small enough to fit into a quart mug*. From this modest beginning, he

became one of the world's greatest scientists. His scientific work was based on differential equations, which he applied successfully to the study of nature.

For example, Newton's second law of motion (force) = (mass) x (acceleration) is a differential equation. When combined with his law of gravitation, Newton's laws of motion enabled him to compute the orbit of the planets, the Moon and comets. He estimated the weight and density of the Sun and Moon. When only the Sun and a single planet are considered, the differential equation obtained by Newton is solvable in an elementary way. As a result, Newton could give a brief derivation of the three laws of Kepler, which Kepler developed through a lifetime of astronomical observations. The problem of determining the motion of two gravitating masses, such as the Sun and a planet, is called two-body problem. The three-body problem is to determine the motion of three gravitating masses, such as the Earth, Sun and Moon. This problem is not elementary at all. Many famous mathematicians have dealt with this problem since Newton.

Many excellent mathematicians have worked on differential equations. Some of the leading names are Leibniz, Daniel Bernoulli, Euler, Laplace, Lagrange, Fourier, Poincare, Picard, Liapunov and Volterra. These later researchers showed that differential equations can be applied not only to Newtonian mechanics, but to a wide variety of scientific fields. For example, differential equations can be applied to fluid flow, propagation of sound waves and the flow of electricity in a cable.

Fourier's book published in 1822, *The Analytic Theory of Heat*, has been called the Bible of mathematical physics. By using differential equations, Maxwell predicted the existence of electromagnetic waves (radio waves) before they were observed experimentally by Hertz. The theory of geometric optics depends on differential equations, and so does the theory of deformation of elastic structures (beams, membranes, etc). Chemical reactions and radioactive decay can also be modelled by using differential equations.

These examples come from such fields as physics, chemistry and engineering, which are sometimes referred to as the *exact sciences*, where the relevant differential equation can be formulated easily.

There are other fields in which differential equations are also essential. Some of these topics are related to economics, ecology and biology. For example, the successful programme of eradicating smallpox is dedicated to Daniel Bernoulli, who set up a differential equation for the progress of an infectious disease in 1760. His work gave a scientific justification for the risk of vaccination.

Another example, when differential equations can be used, is to study the fluctuations of animal populations, such as the Arctic fox or fish in the Adriatic. Both these examples have played a role in the development of mathematical ecology.

Further examples can be given by the mathematics of heart physiology, the transmission equations for nerve impulses, the growth laws of tumours, etc.

To conclude, it would be a mistake to think that differential equations are important only in connection with other fields of science. The *Mathematical Reviews* publishes abstracts of more than 75 new papers on differential equations per month and the number has been increasing from year to year.

If one wants to describe a movement or process by differential equation first one needs a mathematical model. The mathematical description of a phenomenon always entails some simplification. A more exact representation of a physical phenomenon can be described by a more complicated differential equation, which makes it possible to take additional factors into consideration. Providing solutions to some complicated problems or to characterise their properties are still open for the researchers.

AN EXAMPLE: CHAIN CURVE - CATENARY

Take a flexible chain of uniform linear mass density. Suspend it from its two ends.

What is the curve formed by the chain?

Galileo Galilei said that it was approximated by a parabola. This time Galileo was not correct. This curve is called a catenary. Let us determine the shape of the chain.

Let us denote by P an arbitrary point on the chain and the point on the chain located at the origin of the Cartesian coordinate system is denoted by O . Moreover, let the low point of the chain be the origin. Take a part of a chain in the interval between O and P . It is assumed that the chain is at rest. That means that the net force on it is zero. There are three forces acting on the chain interval OP (Figure 1). One force is the tension on the chain to the left. Call this force \vec{H} . It acts in a horizontal direction to the left. There is also a force on the chain to the right, exerted at point P . This force is called \vec{T} . Its direction is what needs to be determined, because that will describe the slope of the chain at P .

The third force is the weight on the chain, \vec{W} . Since this is a gravitational force, its direction can only be vertical and downward.

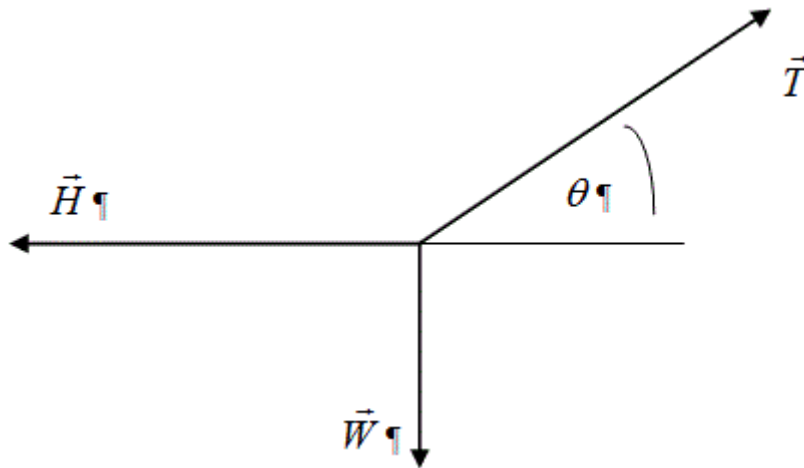


Figure 1: Three forces.

One can see in Figure 1 that the direction of \vec{T} , which gives the slope at P . Force \vec{T} has a horizontal component for which:

$$|\vec{T}| \cos \theta = |\vec{H}| \quad (1)$$

and for the vertical component:

$$|\vec{T}| \sin \theta = |\vec{W}|. \quad (2)$$

That means that:

$$\tan \theta = \frac{dy}{dx} = \frac{W}{H}, \quad (3)$$

where W and H denote the magnitudes for \vec{W} and \vec{H} , respectively, and they are not vectors.

The chain carries only its own weight. That weight is proportional to the length of the chain between O and P .

Let us denote by s the length of the chain, where $w [N/m]$ is the weight per unit length of the chain, then:

$$W = ws. \quad (4)$$

H is clearly a constant. It is the tension at point O , which is the same no matter what point is chosen for P . W is the weight on the chain between O and P .

Is it possible to show how can it be determined?

Again, the weight is proportional to the length of the chain between O and P . It cannot be computed by simply multiplying the horizontal distance by a constant. Hence the length of the chain between O and P must be calculated.

Let y be the height of the chain. Again, w represents the linear weight density, it is weight per length of chain. If s denotes the length of the chain between O and P , this means that the weight of the chain between O and P must be ws .

One may recall this formula for the length of a curve on the interval (a, b) :

$$\int_a^b \sqrt{1 + y'^2} dx. \quad (5)$$

One can use it to find s , the curve length on the interval $(0, x)$:

$$s = \int_0^x \sqrt{1 + y'^2} dt. \quad (6)$$

The derivative dy/dx , is itself a function of s :

$$\frac{dy}{dx} = \frac{W}{H} = \frac{ws}{H} \quad (7)$$

and

$$s = \frac{H}{w} \frac{dy}{dx}. \quad (8)$$

Making the substitution for s in the integration equation above, one obtains:

$$\frac{H}{w} \frac{dy}{dx} = \int_0^x \sqrt{1 + y'^2} dt, \quad (9)$$

and, further:

$$\frac{dy}{dx} = \frac{w}{H} \int_0^x \sqrt{1 + y'^2} dt. \quad (10)$$

Differentiate both sides of the equation with respect to x :

$$\frac{d^2 y}{dx^2} = \frac{w}{H} \sqrt{1 + y'^2}, \quad (11)$$

or

$$y'' = \frac{w}{H} \sqrt{1 + y'^2}. \quad (12)$$

which is a second order nonlinear ordinary differential equation.

Substituting $y' = p(x)$ one gets the following first order differential equation:

$$\frac{dp}{dx} = \frac{w}{H} \sqrt{1 + p^2} \quad (13)$$

and separating the variables, one obtains:

$$\frac{dp}{\sqrt{1 + p^2}} = \frac{w}{H} dx. \quad (14)$$

Let us take the integral of both sides:

$$\int \frac{dp}{\sqrt{1 + p^2}} = \int \frac{w}{H} dx, \quad (15)$$

and apply the primitives to both sides:

$$\operatorname{ar sinh} p = \frac{w}{H} x + C \quad (16)$$

with integration constant C . Hence:

$$y' = p = \sinh\left(\frac{w}{H} x + C\right), \quad (17)$$

and

$$y = \int \sinh\left(\frac{w}{H} x + C\right) dx = \frac{H}{w} \cosh\left(\frac{w}{H} x + C\right) + D, \quad (18)$$

which gives the general solution to the second order differential equation.

In order to determine constants C and D one can apply the initial conditions formulated at the low point of the chain O :

$$y(0) = 0, \quad y'(0) = 0.$$

From condition $y'(0) = 0$ it follows that $C = 0$ and from $y(0) = 0$, one obtains:

$$0 = \frac{H}{w} \cosh 0 + D, \quad D = -\frac{H}{w}.$$

This means that the shape of the chain can be formulated as:

$$y = \frac{H}{w} \left[\cosh \frac{w}{H} x - 1 \right]. \quad (19)$$

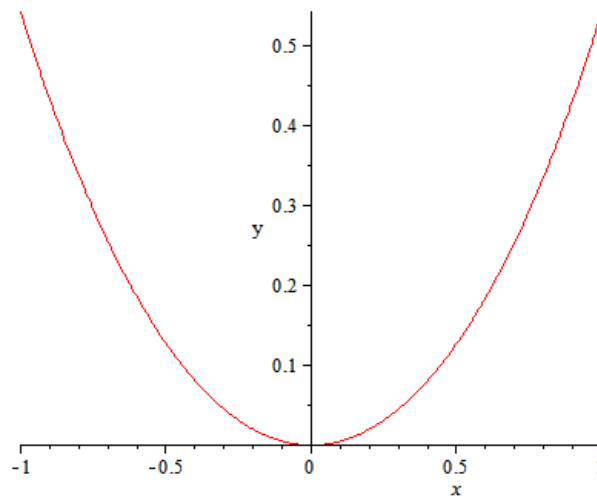


Figure 2: The figure represents the shape of the chain - the catenary.

It should be noted that this curve is very often used in the construction of kilns and gateway arches (See, e.g. *Gateway Arch* in St Luis or Gaudí's *Casa Milá* in Barcelona, Spain).

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