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Growth processes: mathematical basics and projects for students

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ABSTRACT: Growth is a phenomenon occurring everywhere, in nature, in the economy and in society. Mathematical models are hidden behind the different growth processes. After explaining the mathematical basics in the context of growth and after introducing growth models and some of their applications, it is illustrated how teams of students can start a lot of project work containing mathematics, modelling and simulation. During this work, students can learn a lot about mathematics and its relation to practical problems, about the gap between models and reality, about problem solving in teams and about the translation of solutions into software products.

Keywords: Growth, growth models, natural growth, logistic growth, ordinary differential equations of first degree, dynamics, steady states, stability, projects in mathematical education

MATHEMATICAL BASICS AND GROWTH AS PROJECT WORK

Teachers are confronted with big challenges in mathematical education:

- The importance of mathematics for higher education and the professions is scrutinised.
- Many students enter the university with low mathematical skills and low motivation for learning more mathematics.
- There are demands from the public to meet the aspirations of students.
- The new media offer new forms of learning mathematics.

One demand is to teach mathematics in close relation with applications. An interesting opportunity is to explain mathematical basics against the background of growth processes. A lot of mathematical growth models are already available. New ones can be created. Growth is investigated showing the benefit of mathematics and motivating students to learn it. At the same time, aspects of modelling and simulation are included, demonstrating the interplay of different disciplines for the solution of actual problems. Problems are solved by algorithms on a computer.

Growth is a general phenomenon. Nevertheless, it seems to be clear that growth is limited to earlier or later. Therefore growth is also an emotive word in the political discussion [4].

One idea of modern teaching is breaking up the classical frontal teaching by using more cooperative forms. Another idea is to use modern means, such as computers for computation and exploration [1][6] or the Internet for information and teaching material. Occasionally, projects are introduced at the engineering faculty of Wismar University, where teams of students work on practically relevant problems, which require a mathematical background, guided by staff members. The focus is put on problem solving. Mathematical software, such as MATLAB is used as an additional tool. In the past, good experiences were made with such topics as computerised tomography (CT), oscillators in engineering or predator-prey models. The level of mathematics can vary on a wide scale and has to fit to the pre-knowledge of students. If projects are included in the first year mathematical education, a revision of syllabus might be necessary. Another possibility is to offer projects in later semesters as optional courses. Then, integration of engineering subjects and cooperation with colleagues from engineering is more fruitful. A new project about growth processes needs only mathematical basics and calculus, but it can also be extended to include such subjects as ordinary differential equations, numerical mathematics, probability and statistics, as well as computer mathematics [8]. Since all these subjects occur in the first year course of engineering mathematics, at most the order of content has to be changed. Some of the usual topics can be cancelled to avoid overloading of the course.

GROWTH FUNCTIONS

A *process* is described here by a real smooth (sufficiently often differentiable) function $x = x(t)$, where $t \in T = [t_0, t_e[$ denotes the time. Usually, an initial value of the process $x(t_0) = x_0$ is given. In most cases $t_0 = 0$ and $t_e = +\infty$ is assumed. A function $x(t)$ represents *growth* if it is monotone increasing. The counterpart of monotone increasing functions are monotone decreasing functions $x(t)$. They describe processes of decay, decrease, shrinking or reduction. Growth can be *bounded*, *unbounded* or even *explosive*. Typical cases of unbounded growth are *polynomial* and *exponential* functions as::

$$x(t) = t^n \ (n \geq 1, t \geq 0), \quad x(t) = e^t - 1 \ (t \geq 0) \,. \tag{1}$$

Examples of explosive growth are time functions with poles as:

$$x(t) = \frac{t}{1-t} \ (0 \leq t < 1), \quad x(t) = \tan t \quad \left(0 \leq t < \frac{\pi}{2}\right). \tag{2}$$

Typical examples of bounded growth are functions as:

$$x(t) = \frac{t^2}{t^2+1} \ (t \geq 0), \quad x(t) = \frac{e^t - 1}{e^t} \ (t \geq 0), \quad x(t) = \frac{2}{\pi} \tan^{-1} t \ (t \geq 0) \,, \tag{3}$$

where the limit $\lim_{t \to +\infty} x(t) = C$ is said to be the *capacity* of the process. The capacity is 1 in all three examples (3).

Further investigations and classifications can be made using the differential calculus. A growth function $x(t)$ is characterised by a positive (non-negative) time derivative (velocity, *growth rate*) $x'(t) \geq 0 \ (t \in T)$, while $x'(t) \leq 0 \ (t \in T)$ holds for decrease. In the special case $x'(t) = 0 \ (t \in T)$, the process x is *stationary* (constant). It is a *steady state*. The growth is said to be *progressive*, if the growth rate $x'(t)$ is monotone increasing (type 1). Therefore, the second derivative (velocity) is positive (non-negative): $x''(t) \geq 0 \ (t \in T)$. The corresponding curve segment is *convex*. The growth is called *digressive* for $x''(t) \leq 0 \ (t \in T)$ (type 2). The curve segment is *concave*. Both features can also be mixed. Then, there is a time t_u with $x''(t_u) = 0$ (turning point at t_u, type 3). The type depends also on the time interval T. It is possible that x is of type 3 in the range of definition, but of type 2 in a subinterval T ($t_u \notin T$).

Example: Who is familiar with stochastics recognises an interesting cross connection: all *probability distribution functions* $x(t)$ vanishing outside T are examples for bounded growth with capacity 1.

DIFFERENTIAL EQUATIONS OF FIRST ORDER

Time processes $x(t)$ can be described mathematically by differential equations. Here only equations of first order:

$$x'(t) = f(t, x(t)) \quad (t \in T) \tag{4}$$

are considered, where f is a function of two variables controlling the velocity of change. Generally, there is a whole family of solutions, containing an arbitrary constant C. The graphical representation can be supported by drawing a *direction field* [2]. If f is positive for a solution x in the interval T, then, x is monotone increasing and, therefore, a growth function on T. Including an *initial value* $x(t_0) = x_0$, the solution x is (under some natural assumptions) unique. If f does not explicitly depend on the time t, the differential equation is called *autonomous*. Then, it holds:

$$x'(t) = \frac{dx}{dt} = f(x), \quad x = x(t) \quad (t \in T) \,. \tag{5}$$

This type can be solved by *separation of the variables* and integration. Then, it is:

$$G(x) = \int \frac{dx}{f(x)} = t + C \,, \tag{6}$$

where G is an antiderivative of the reciprocal of f and C is the constant of integration. Although G generally exists, it can be a problem to find it analytically. Assume, e.g. that f is a polynomial. Then, the integrand is a rational function which can be split into partial fractions if the zeros of f are known. But zeros of polynomials have to be determined numerically in many cases. Hence, the final solution formula often holds only approximately.

In the following $T = [0, +\infty[$ is supposed. The initial condition $x(0) = x_0$ determines the constant by $C = G(x_0)$. If G is invertible, the implicit solution formula can be written explicitly:

$$x(t) = G^{-1}(t + C) = G^{-1}(t + G(x_0)) . \tag{7}$$

However, often this will not be the case. Consequently, numerical methods come in to solve differential equations. MATLAB is professional software to do so [1][6]. MATLAB supplies not only the approximate solution but also a graphical representation. If students are not acquainted with such methods, it is useful to start with elementary methods as Euler, Heun or the classical Runge-Kutta method and to compare the accuracy of the corresponding solutions. Such simple routines can be written by students themselves. But, if high accuracy is needed, the more sophisticated methods of MATLAB should be applied.

STEADY STATES AND STABILITY

Considering the dynamical behaviour of a growth process *steady states* (equilibriums) are of special interest. Steady states x_s occur for vanishing velocity x'. For the type $x' = f(x)$ these states are zeros of f, that means $f(x_s) = 0$. Eventually, numerical methods are necessary to determine these zeros at least approximately. Two different kinds of steady states are possible: stable ones attracting the process and unstable ones repelling it. For a simple analysis, it is sufficient to study the process behaviour locally in a small neighbourhood of the steady state. If x_s is a simple zero of f, that is $f'(x_s) \neq 0$, then it is sufficient to approximate $f(x)$ around x_s by using the tangent:

$$x' = f(x) \approx f(x_s) + f'(x_s) \cdot (x - x_s) = f'(x_s) \cdot (x - x_s) . \tag{8}$$

Then, $f'(x_s) < 0$ (negative slope) means that small changes of x away from x_s lead to a return. Hence, x_s is *stable*. On the other hand, if $f'(x_s) > 0$ (positive slope), then, the process is running away from x_s. Hence, x_s is *unstable*. So, it is easy to check the stability of steady states. But, it can happen that $f'(x_s) = 0$. Then, the Taylor expansion of f must be considered up to the first non-vanishing higher derivative. For simplicity, let us assume $f''(x_s) \neq 0$ in this case. Then, using quadratic approximation, it holds:

$$x' = f(x) \approx f(x_s) + f'(x_s) \cdot (x - x_s) + \frac{1}{2} \cdot f''(x_s) \cdot (x - x_s)^2 = \frac{1}{2} \cdot f''(x_s) \cdot (x - x_s)^2 . \tag{9}$$

Hence, the sign of x' is the sign of $f''(x_s)$. The steady state is *indefinite*. Supposing $f''(x_s) > 0$ the process is attracting for $x < x_s$ and repelling for $x > x_s$. A corresponding result is true for $f''(x_s) < 0$. The stability depends on the branch of function which is picked up by the initial value x_0.

MODELS OF LINEAR TYPE

First, the simple model class:

$$x'(t) = b \cdot x + c \quad (b, c \in R, b \neq 0) \tag{10}$$

with parameters b, c is considered. This is a linear differential equation with constant coefficients. There is one steady state, namely $x_1 = -c/b$. With the notation $P(x) = b \cdot x + c$ it holds $P'(x) = P'(x_1) = b$. Hence x_1 is stable only for $b < 0$. The solution of (10) is:

$$x(t) = C \cdot e^{b \cdot t} - \frac{c}{b} = \left(x_0 + \frac{c}{b} \right) \cdot e^{b \cdot t} - \frac{c}{b} . \tag{11}$$

First $b > 0$ is supposed. Then, x represents *exponential growth*. The classical case results for $c = 0$. This behaviour is typical if growth forces can freely spread out without any restrictions. If a colony of bacteria with size $x(t)$ (number of bacteria) is considered, then, it seems reasonable that the growth rate $x'(t)$ is proportional to the size. Malthus assumed such a model for human earth population and predicted in 1798 a worldwide catastrophe in the near future. This forecast turned out to be false. The earth population did not develop according to this model, at least regarding large time periods [8]. Now, let be $b < 0$. Then, the process is bounded, namely $\lim_{t \to +\infty} x(t) = x_1$. If additionally $c = 0$, then, *exponential* or *natural decay* occurs.

If the model has the form:

$$x'(t) = q \cdot (K - x) = q \cdot K \cdot \left(1 - \frac{x}{K}\right) \quad (q, K > 0, b = -q, c = q \cdot K\ ), \tag{12}$$

then, the solution is:

$$x(t) = K \cdot \left(1 - e^{-q \cdot t}\right) + x_0 \cdot e^{-q \cdot t}, \quad \lim_{t \to +\infty} x(t) = x_1 = K\ . \tag{13}$$

Here, the growth rate x' is proportional to the difference $K - x$. For $0 < x_0 < K$ this is a bounded growth process of type 2 (digressive, concave).

Examples: Suppose that information is spread by media in a region with *K* peoples. A simple assumption is that the number *x*(*t*) of people which have the information at time *t* fulfils nearly the above model. The same is true for saturation processes as learning, compensation processes and diffusion in cells.

SIMPLE MODELS OF NONLINEAR TYPE

Now, the model class:

$$x'(t) = a \cdot x^2 + b \cdot x + c \quad (a, b, c \in R, a \neq 0) \tag{14}$$

with parameters *a, b, c* is investigated. With the notations $P(x) = a \cdot x^2 + b \cdot x + c$ and $D = b^2 - 4a \cdot c$ the quadratic polynomial *P*(*x*) has:

1. two different real zeros x_1 and x_2 for $D > 0$, where $x_1 < x_2$ and $P(x) = a \cdot (x - x_1) \cdot (x - x_2)$,
2. a real doubled zero x_1 for $D = 0$, where $P(x) = a \cdot (x - x_1)^2$,
3. no real zero for $D < 0$.

In case 1, there are two steady states:

$$x_1 = \frac{1}{2a} \cdot \left(-b - \sqrt{D}\right), \quad x_2 = \frac{1}{2a} \cdot \left(-b + \sqrt{D}\right) \tag{15}$$

representing the constant solutions $x = x_1$ and $x = x_2$. Since the graph of *P*(*x*) is a parabola, x_1 is stable and x_2 is unstable for $a > 0$, while x_1 is unstable and x_2 is stable for $a < 0$. In case 2, there is only one steady state $x_1 = -b/(2a)$ and one constant solution $x = x_1$. Here, it is $P'(x_1) = 0$ and $P''(x_1) = 2a$ such that the steady state is indefinite. In case 3, there is no steady state. By separation of the variables, partial fraction decomposition and integration the following solutions are obtained:

$$\begin{aligned}
&1.\ x(t) = \frac{x_1 - C \cdot x_2 \cdot e^{a \cdot (x_1 - x_2) \cdot t}}{1 - C \cdot e^{a \cdot (x_1 - x_2) \cdot t}} = x_2 - \frac{x_2 - x_1}{1 - C \cdot e^{a \cdot (x_1 - x_2) \cdot t}} \quad \left(t \neq \frac{\ln C}{a \cdot (x_2 - x_1)}\right) \\
&2.\ x(t) = x_1 - \frac{1}{a \cdot t + C} \quad \left(t \neq -\frac{C}{a}\right) \\
&3.\ x(t) = \frac{1}{2a}\left[-b + \sqrt{|D|} \cdot \tan^{-1}\left(\frac{1}{2}\sqrt{|D|} \cdot t + C\right)\right]
\end{aligned} \tag{16}$$

Here, *C* is the arbitrary integration constant. Putting $x(0) = x_0$, it holds:

$$1.\ C = \frac{x_1 - x_0}{x_2 - x_0}, \quad 2.\ C = \frac{1}{x_1 - x_0}, \quad 3.\ C = \tan \frac{2a \cdot x_0}{\sqrt{|D|}}\ . \tag{17}$$

In case 1, the function can be defined everywhere, namely for $C \leq 0$. This is true, if $x_1 < x_0 < x_2$. Further:

$$\begin{aligned}
&1.\ \lim_{t \to +\infty} x(t) = x_1 \ \ for\ a > 0, \quad \lim_{t \to +\infty} x(t) = x_2 \ \ for\ a < 0 \\
&2.\ \lim_{t \to +\infty} x(t) = x_1 \\
&3.\ \lim_{t \to +\infty} x(t) = \frac{1}{2a} \cdot \left(-b + \sqrt{|D|} \cdot \frac{\pi}{2}\right)
\end{aligned} \tag{18}$$

Bounded growth occurs, for example, under the following conditions:

$$1.\, a < 0, \quad x_1 < x_0 < x_2, \quad 2.\, a > 0, \quad x_0 < x_1, \quad 3.\, a > 0\,. \tag{19}$$

Now, let us consider $a < 0, b > 0, c = 0$. This is case 1 and leads to the *logistic differential equation*:

$$x'(t) = b\cdot x - q\cdot x^2 = b\cdot x\cdot\left(1-\frac{x}{K}\right) \quad \left(b>0, q=-a>0, x_1=0,\ x_2=K=\frac{b}{q}>0, C=-\frac{x_0}{x_2-x_0}\right), \tag{20}$$

where $x_2 = K$ is the capacity. The solutions of (20) are:

$$x(t) = \frac{C\cdot K\cdot e^{b\cdot t}}{C\cdot e^{b\cdot t}-1} = \frac{C\cdot K}{C-e^{-b\cdot t}} = K - \frac{K}{1-C\cdot e^{b\cdot t}} \quad \left(t \neq -\frac{\ln C}{b}\right) \tag{21}$$

or, considering the initial value:

$$x(t) = \frac{x_0\cdot K\cdot e^{b\cdot t}}{K+x_0\cdot\left(e^{b\cdot t}-1\right)} = \frac{x_0\cdot K}{x_0+\left(K-x_0\right)\cdot e^{-b\cdot t}} = K - \frac{K\cdot\left(K-x_0\right)}{K+x_0\cdot\left(e^{b\cdot t}-1\right)} \quad \left(t\neq\frac{1}{b}\ln\frac{x_0-K}{x_0}\right). \tag{22}$$

Under the condition $0 < x_0 < K$ *logistic growth* appears. This is the most important type of bounded growth. The logistic differential equation, and its solution, were introduced by the Belgian mathematician and demographer Verhulst in 1836 to model the development of human population (compare with the prediction of Malthus in 1798 (see earlier section). A lot of growth processes with limited capacities can be well approximated by the logistic approach.

Examples: The logistic approach is a simple model for spreading of information in a region with *K* people by passing it on orally starting for instance with $x_0 = 1$ person. This approach is also applied to reproduction of populations with a limited environment or competition situation, spreading of diseases, as well as growth of organisms and plants (see examples in References [2][9]).

Another interesting assumption is $a > 0, b < 0, c = 0$. This is again case 1 with the differential equation rewritten as:

$$x'(t) = a\cdot x^2 - p\cdot x = p\cdot x\cdot\left(\frac{x}{K}-1\right) \quad \left(a>0, p=-b>0, x_1=0, x_2=K=\frac{p}{a}>0, C=-\frac{x_0}{x_2-x_0}\right). \tag{23}$$

The equation is similar to the logistic one, but with opposite sign of the right-hand side. The solution has the form:

$$x(t) = \frac{C\cdot K\cdot e^{-p\cdot t}}{C\cdot e^{-p\cdot t}-1} = \frac{C\cdot K}{C-e^{p\cdot t}} = K - \frac{K}{1-C\cdot e^{-p\cdot t}} \quad \left(t\neq\frac{\ln C}{p}\right). \tag{24}$$

For $x_0 > K$, there is a time $t_e > 0$, where the denominator vanishes and *x* tends over all limits. This means explosive growth. For $0 < x_0 < K$ the solution *x* tends to zero for increasing time *t*. Hence, for initial values $x_0 \approx K$, the behaviour of the process is unpredictable. Small perturbations can completely change the process. Applied to population models the alternatives are doomsday or extinction [2].

SOME INTERESTING DERIVATES

A simple extension of the logistic model is:

$$x'(t) = b\cdot x - q\cdot x^2 - k = b\cdot x\cdot\left(1-\frac{x}{K}\right) - k \quad \left(b,q>0,\ k\geq 0, K=\frac{b}{q}>0\right). \tag{25}$$

The constant *k* can be interpreted as harvesting of the population *x* if $k > 0$. Think of a fish population being caught at a constant rate. Another modification of logistic model is given in [9] by:

$$x'(t) = b\cdot x\cdot\left(1-\frac{x}{M}\right)\cdot\left(1-\frac{m}{x}\right) = -\frac{b}{M}\cdot x^2 + b\cdot\left(1-\frac{m}{M}\right)\cdot x - b\cdot m \quad (b>0, 0<m<M, x\neq 0)\,, \tag{26}$$

where the extra factor in the product considers that some species tend to become extinct if their size falls below a certain minimum *m*. The negative leading coefficient in the quadratic expression shows that it is case 1 of earlier sections. The zeros (steady states) are here *m* and *M*. Bounded growth occurs for $m < x_0 < M$. The equation:

$$x'(t) = b \cdot x - q \cdot x^{1+\delta} = b \cdot x \cdot \left(1 - \left(\frac{x}{K}\right)^{\delta}\right) \quad \left(b, q, \delta > 0, K^{\delta} = \frac{b}{q}\right) \tag{27}$$

is also a generalisation of the logistic model. The classical case arises for $\delta = 1$. There are again two steady states, the unstable $x_1 = 0$ and the stable $x_2 = K$. Growth with capacity K appears for $0 < x_0 < K$.

Example: population size of vertebrates and invertebrates for $\delta > 1$ and $\delta \le 1$, respectively.

The logistic model can be extended by subtracting on the right-hand side of the differential equation a rational function $p(x)$ reflecting a further concrete influence:

$$x'(t) = b \cdot x \cdot \left(1 - \frac{x}{K}\right) - p(x), \quad p(x) = \frac{B \cdot x^2}{x^2 + A^2} \quad (b, A, B > 0, 0 < x(0) = x_0 < K)\,. \tag{28}$$

This equation was used to model outbreak of spruce budworm population x defoliating fires in Canadian forests. The additional function $p(x)$ models predation by birds [5]. Here, 0 is an unstable steady state. Depending on the parameters there are one or three positive steady states. In the latter case, the steady states characterise in natural order the stable *refuge*, the unstable *transit* and the stable *outbreak equilibrium*. A reasonable strategy should be to minimise the damage of fir by the spruce budworm that is to prevent the outbreak equilibrium. An important question is to find out the relation between practical steps and parameter change.

A further interesting growth model was proposed by Gompertz (1779-1865) [1]:

$$x'(t) = a \cdot e^{-\alpha \cdot t} \cdot x(t) \quad (a > 0, \alpha > 0)\,. \tag{29}$$

This equation is not autonomous. The relative growth rate is a decreasing time function. If x is the size of a population, this considers the lower growth potential of members with progressive age. Bounded growth is obtained for $x_0 > 0$.

Examples: growth x of volume of hard tumours in medicine, mass x of fish in water [1].

DATA FIT

If experimental data (t_i, x_i) of a growth process are given, then, they can be approximated by a growth function $x=x(t)$, where $x_i \approx x(t_i)$ ($i = 0,1,...,n$). If the type of growth is known or estimated, then, often $x(t)$ can be determined in a corresponding class of functions characterised by $m+1$ parameters (including the initial value). Methods are rough estimation, interpolation (eventually, by ignoring some data) or approximation (by using an optimisation criterion, e.g. least squares). MATLAB can help in difficult cases.

Exercise: The growth of earth population should be fit to different models starting with the year t_0. The growth can be assumed to be exponential, super-exponential and logistic, respectively:

$$x(t) = x_0 \cdot e^{b \cdot t}, \quad x(t) = x_0 \cdot e^{b \cdot t + c \cdot t^2}, \quad x(t) = \frac{x_0 \cdot K \cdot e^{b \cdot t}}{K + x_0 \cdot \left(e^{b \cdot t} - 1\right)}\,. \tag{30}$$

The approaches (30) contain two, three and again three parameters. Hence, at least so many data are needed to get enough equations for the unknown parameters. The initial value is determined by the first data value.

If the general solution is not known, then, it is also possible to estimate the parameters in the differential equation from the data (for the logistic equation see Reference [2]).

ACTIVITIES AND REACTIONS OF STUDENTS

The topic of growth offers many options for project work in higher education where general questions in society and science can be connected with a theoretical background using mathematical basics. A selection of topics is given:

- Political and philosophical questions connected with growth processes.
- Study of function families representing different qualities of growth.
- Collection of statistical data concerning real growth processes; data fit in appropriate function families.
- Modelling of growth processes by differential equations of first order, identifying the meaning of parameters in the real context.

- Investigating growth processes by application of calculus (properties, type of growth).
- Calculating steady states of growth processes and their type (analytical and numerical methods).
- Solving differential equations of first order (analytical and numerical methods).
- Investigating growth models using different parameters and solution methods, comparing results.
- Collecting and solving theoretical and practical exercises based on mathematical models of growth processes.
- Use of MATLAB to produce problem solutions applying graphic, numeric and symbolic means; comparing with results obtained by manual calculation or given in the literature.

There are many textbooks containing appropriate material concerning mathematical tools and application oriented exercises [1][2][9]. Up to now, the project of growth processes was only tested in an optional course. The reactions of students were very positive in general. During the evaluation, the following answers were obtained (translation from the German language):

- The project is innovative and increases my motivation to learn mathematics.
- The project involves interdisciplinary thinking, problem solving, modelling and simulation. Hence, the practical solution to problems is learnt.
- The project stimulates discussion about general problems of society. So, it contributes to the awareness of own responsibility.
- The project is more challenging and activating than classical lecturing.
- The project supports independent and responsible work as well as cooperation.
- I can use computer software and resources from the Internet. This extends my knowledge considerably.
- I like team working with my friends.
- The project was interesting, but I would prefer projects which are concerned with problems of electrical engineering.

The majority of students benefited from the project (> 90%, motivation, knowledge, performance). In addition, staff derived some benefit. A lot of additional material was collected by students.

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BIOGRAPHY

Dieter Schott, starting as a Professor of Numerical Mathematics and Technical Mechanics, is now a Professor of Mathematics in the Department of Electrical Engineering and Computer Science at the University of Wismar, Germany. He graduated from the University of Rostock, Germany, as a mathematician in 1972. He received there a Doctorate in 1976 and the Habilitated Doctor's degree in 1982 in the field of mathematics. Later, he worked at the Universities of Güstrow and Rostock. There, he was engaged in the education of scientists, teachers and engineers. His numerous publications are mainly related to the field of Numerical Analysis. Since 1994, he has been teaching engineering students in Wismar. Professor Schott promotes the use of computers in the lectures of basic sciences and the introduction of international degree programmes. He was director of the Gottlob-Frege-Centre at the University of Wismar from its foundation in November 2000 up to the end of 2010. This centre coordinates basic science education. The aim is to strengthen this education, to bring it up to date and to make it global. Professor Schott organises regional and international conferences about didactics of engineering mathematics. He is the author of the German textbook *Ingenieurmathematik mit MATLAB*.