

Design and development of a training apparatus for strain measurement

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ABSTRACT: It is difficult to determine stresses for a structure, which has a complex shape or which has several loads applied to it. This study explains the development of a training support tool to help students to understand the relation between stress and strain, elastic modulus, the deflection concept and strain measurement principles using a strain gauge. For that purpose, an experiment was designed to determine the magnitude and direction of principal stress at any point where the rosette gauge was attached. It is concluded that the tester can be used widely in the stress analysis of machine and structure as Mohr's circle is used to analyse the magnitude and direction of principal stress under a combined loading condition.

Keywords: Strain measurement, strain gauge, Mohr's circle, principal stress

INTRODUCTION

It is difficult to determine stresses for a structure which has a complex shape or which has several loads applied to it. Failure of a structural member or machine parts can be caused by excessive normal stress or shear stress and, thus, it is important to determine the maximum principal stress and maximum shear stress at any point concerned. Hook's law is at work between stress and strain when an object has elastic deformation caused by external force and, therefore, an experimental method is used that measures strain and subsequently, calculates stress. Among many experimental methods, ones using a strain gauge are relatively most convenient and easy to use, and consequently are widely applied.

This study is intended to develop a training support tool to help students to understand the relation between stress and strain, elastic modulus, the deflection concept and strain measurement principles using a strain gauge. To that end, an experiment was implemented that determined normal stress at an attached point with a strain gauge, and deflection on the end of a cantilever beam after applying normal load to the end of the beam. In addition, an eccentric load was applied to the end of the cantilever beam. In other words, a bending load and torsional load were applied simultaneously to analyse the stress state under a combined loading condition. Based on this, an experiment was designed to determine the magnitude and direction of principal stress at any point where the rosette gauge was attached.

MAIN CONSIDERATIONS ABOUT TESTING DEVICES

1. A tester should be developed, which enables students to learn a way of using a strain gauge, by personally attaching a uniaxial strain gauge.
2. An experiment should be implemented for the purpose of determining the elastic modulus of a beam based on experimental strain and theoretical stress after applying a load to the end of the beam.
3. A tester should be designed, which contributes to understanding the relation between the load P and deflection δ of a beam by the application of changing loads to the end of the cantilever beam.
4. A tester should be designed for the purpose of determining the magnitude and direction of principal stress by using Mohr's circle, based on strain detected in a triaxial rosette strain gauge attached to any point of a cantilever beam.
5. For this test, applied load control, load measurement, displacement measurement and strain measurement system are mainly necessary and a portable tester size should be adopted.

CONCEPT OF TESTING DEVICE DESIGN

As seen in Figure 1, after selection of an L-shaped cantilever beam, the narrow ending part is fixed while load is applied to the opposite end and strain is measured by using a triaxial strain gauge attached to position A. The beam is designed so that bending and torsion occur simultaneously with application of load to position D, so as to measure strain with a rosette gauge. Additionally, a uniaxial strain gauge is attached to position B by students and different loads are applied to position C to measure experimental strain (ε). And, the elastic modulus (E) of the beam is decided from Hook's law ($\sigma = E\varepsilon$), and the theoretical stress:

$$\left(\sigma = \frac{My}{I}\right). \quad (1)$$

Also, deflection at the end part is measured through the displacement gauge with different loads being applied to position C, to make comparisons between the experimental formula and the theoretical formula:

$$\left(\delta = \frac{PL^3}{3EI}\right), \quad (2)$$

about the relation between load and deflection.

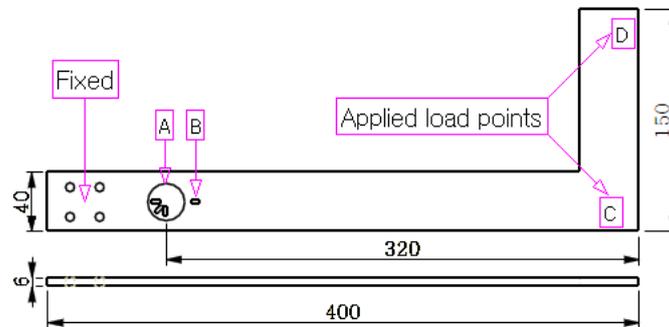


Figure 1: Shape and size of L-shaped cantilever beam and rosette gauge A attachment point.

STRAIN, DISPLACEMENT AND LOAD MEASUREMENT SYSTEM DESIGN

In order to apply load, an adjustment is made by turning the micrometer head 3, seen in Figure 2, instead of a dead weight. And, the load measured at the load-cell 1 can be identified through the load-cell indicator 8. For the displacement measured at the displacement gauge 4, seen in Figure 2, the deflection is identified through the displacement gauge indicator 9; thus, allowing it to be compared with the theoretically calculated deflection of the cantilever beam. Also, the strain amplifier 7 (National Instrument Co. model, USB DAQ 9237), seen in Figure 2, is used to measure the strain. Figure 3 displays a setup for strain measurement using a LabVIEW program and monitoring system for determining the direction and magnitude of principal stress and measured strain. Table 1 includes a summary of main parts.

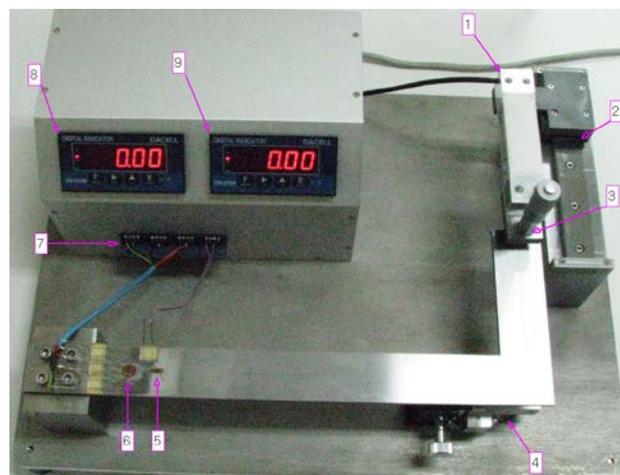


Figure 2: Components of the developed tester.

Table 1: Main parts.

Part Number	Description
1	Load-cell
2	LM-guide
3	Micrometer head
4	Displacement gauge
5	Uniaxial strain gauge
6	Triaxial rosette strain gauge
7	Strain amp
8	Load-cell indicator
9	Displacement gauge indicator

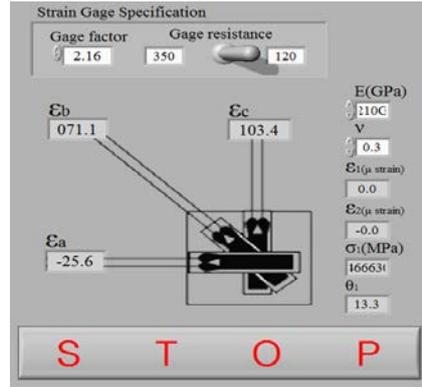


Figure 3: Front panel for monitoring the magnitude (σ_1) and direction (θ_1) of the principal stress and strains ($\epsilon_a, \epsilon_b, \epsilon_c$) from a rosette gauge. Also included are: setup for the strain measurement such as gauge factor; and gauge resistance.

EXPERIMENTAL RESULTS ANALYSIS

The user interface, shown in Figure 3, is written in LabVIEW. The interface includes a real-time plot of the applied load. The triaxial strain gauge 6 is attached at 45 degrees to the L-shaped specimen seen in Figure 2 and, then, the micrometer head is used to apply the load of 39.2N at a point 300mm longitudinally and 105mm laterally away from the gauge. And, the strain originating from the gauge is measured to determine principal strain (ϵ_1, ϵ_2). Based on this, principal stress σ_1 and principal stress direction (θ_1) are calculated, to be compared with the magnitude and direction of principal stress that has been theoretically calculated. The strain seen in Figure 4 was obtained by implementing the experiment. Meanwhile, elastic modulus E and Poisson's ratio ν of the specimen were determined through the experiment: $\nu = 0.3$ and $E = 190.5\text{GPa}$.

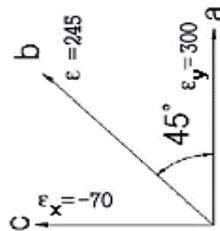


Figure 4: A state of plane strain at the centre of rosette gauge with $P = 39.2\text{N}$ (unit: microstrain).

Principal strain (ϵ_1, ϵ_2) under the state of plane strain is decided through Mohr's circle (Figure 5).

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{\epsilon_c + \epsilon_a}{2} \pm \sqrt{\left(\frac{\epsilon_c - \epsilon_a}{2}\right)^2 + \left(\frac{2\epsilon_b - \epsilon_a - \epsilon_c}{2}\right)^2}$$

$$= \frac{-70+300}{2} \pm \sqrt{\left(\frac{-70-300}{2}\right)^2 + \left(\frac{2 \times 245 - 300 + 70}{2}\right)^2}$$

$$= 407, -145$$

$$\therefore \varepsilon_1 = 407 \mu S, \varepsilon_2 = -145 \mu S$$

The angle Φ in Figure 5 can be determined as follows:

$$\sin \phi = \frac{\frac{\gamma_{xy}}{2}}{R} = \frac{260}{292} = 0.455$$

$$\phi = 26^\circ$$

$$\therefore \theta = \frac{\phi}{2} = 13^\circ$$

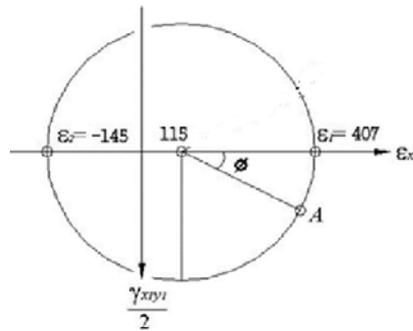


Figure 5: Mohr's circle of strain with the principal strains and maximum in-plane shear strains for the centre of a rosette gauge.

Finally, the direction of principal strain ε_1 ($= 407 \mu S$) will be 13 degrees in a counterclockwise direction from the strain ε_a ($= 300 \mu S$) axis, as shown in Figure 6a.

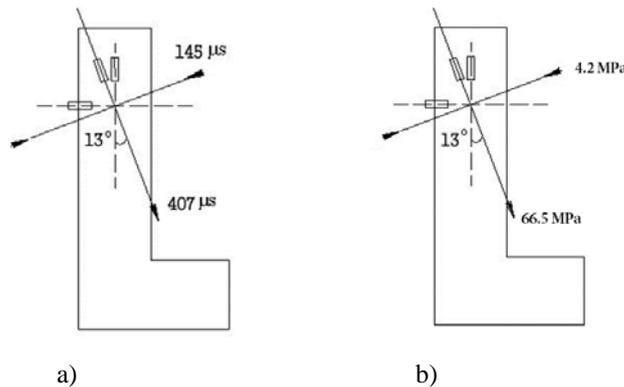


Figure 6: a) The principal strains; and b) the corresponding principal stresses of the testing specimen with $P = 39.2N$.

The principal in-plane stresses can be calculated from the in-plane principal strains as follows:

$$(\sigma_1)_{xy} = \frac{E}{1-\nu^2} (\varepsilon_1 + \nu \varepsilon_2) = \frac{150.5 \times 10^3}{1-0.3^2} [407 + 0.3 \times (-145)] \times 10^{-6} = 60.1 MPa$$

$$(\sigma_2)_{xy} = \frac{E}{1-\nu^2} (\varepsilon_2 + \nu \varepsilon_1) = \frac{150.5 \times 10^3}{1-0.3^2} [-145 + 0.3 \times (407)] \times 10^{-6} = -4.2 MPa$$

Finally, the principal in-plane stresses is as shown in Figure 6b. If one orders the principal stresses, including $\sigma_z = 0$, in the order $\sigma_1 > \sigma_2 > \sigma_3$ one obtains $\sigma_1 = 66.5 MPa$, $\sigma_2 = 0$, $\sigma_3 = -4.2 MPa$

THEORETICAL STRESS ANALYSIS METHODS AND COMPARISON

The theoretical magnitude and direction of principal stress can be determined as follows.

Stress σ influenced by bending momentum M ($39.2\text{N} \times 300\text{mm}$) can be determined as follows:

$$\sigma = \frac{My}{I} = \frac{39.2 \times 300 \times 3}{\frac{40 \times 6^3}{12}} = \frac{35280}{720} = 49.0\text{MPa}$$

Shear stress τ influenced by torque T ($39.2 \times 105\text{mm}$) can be determined as follows:

$$\tau = \frac{T}{c_1 ab^2} = \frac{39.2 \times 105}{0.305 \times 40 \times 6^2} = \frac{4116}{439.2} = 9.4\text{MPa}$$

Wherein, c_1 referring to the torsion constant for a rectangular bar, has the constant of 0.305 [1] with the relation of a/b as seen in Figure 7. Finally, the stress state at the centre of the rosette is displayed in Figure 8.

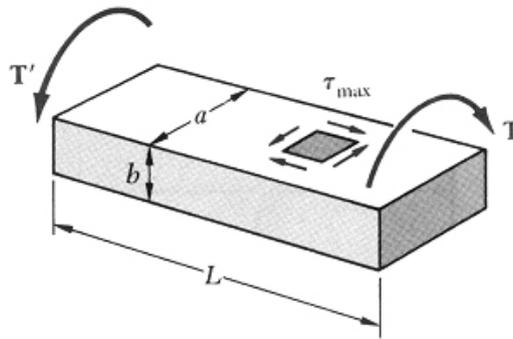


Figure 7: Torsion constant for rectangular bar.

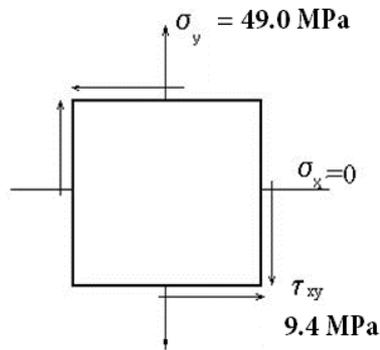


Figure 8: The given state of plane stress at a point with $P = 39.2\text{N}$.

With regards to the stress state in Figure 8, the magnitude and direction of principal stress σ_1 , σ_2 are determined as $\sigma_1 = 50.7\text{MPa}$, $\sigma_2 = -1.7\text{MPa}$ by using Mohr's circle, see in Figure 9.

The angle Φ in Figure 9 can be determined as follows:

$$\sin \phi = \frac{9.4}{26.2} = 0.36, \phi = 21^\circ$$

$$\therefore \theta = \frac{\phi}{2} = 10.5^\circ$$

Finally, the direction of principal stress σ_1 ($= 50.7\text{MPa}$) can be 10.5 degrees in a counterclockwise direction from the stress σ_y ($= 49\text{MPa}$) axis, as shown in Figure 10.

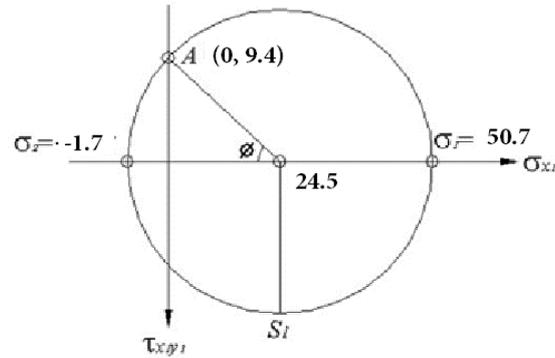


Figure 9: Mohr's circle of stress with the principal stresses and maximum in-plane shear stresses.

In this experiment, an eccentric load of $P = 39.2\text{N}$ was applied to a point 300mm and 105mm away from the centre of the rosette gauge. In other words, the direction and magnitude of principal stress relative to the centre of the rosette gauge were tested under a combined loading condition. Table 2 summarises the experimental and theoretical analysis results.

The magnitudes of maximum principal stress for experiment and theory were 66.5MPa and 50.7MPa, respectively, indicating that the experimental magnitude had a 31.2% bigger error, compared to the theoretical one. In addition, the directions of maximum principal stress in experiment and theory were 13 degrees and 10.5 degrees, respectively, indicating that the experimental magnitude had a 23.8% bigger error than the theoretical one. The installation of the rosette gauges may partially incur this error.

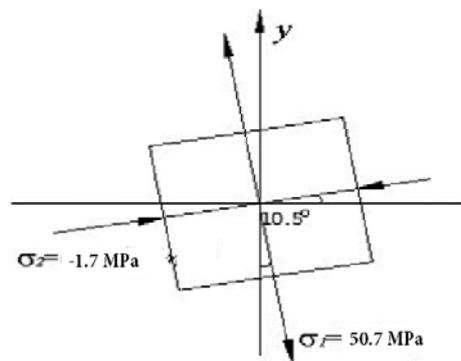


Figure 10: The principal stresses at the centre of the rosette gauge on the testing specimen with $P = 39.2\text{N}$.

Table 2: Experimental and theoretical results by an eccentric force $P = 39.2\text{N}$.

	σ_1 (MPa)	σ_2 (MPa)	θ_1 (degree)
Theory	50.7	-1.7	10.5
Experiment	66.5	-4.2	13
Error (%)	31.2	-	23.8

CONCLUSION

This tester is thought to help students to effectively understand the principles of stress, elastic modulus, strain and deflection, which are important concepts in material mechanics, therefore, contributing to a maximum learning effect. Additionally, it is concluded that the tester can be used widely in the stress analysis of machine and structure as Mohr's circle is used to analyse the magnitude and direction of principal stress under a combined loading condition.

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BIOGRAPHIES



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