Volume 20, Number 1, 2018

Global Journal of Engineering Education

Comparative study on the boundary element method meshing techniques applied within the boundaries of a 3D computer-aided design system

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ABSTRACT: The integration of mechanical calculation tools with computer-aided design (CAD) in the design process within a computer-aided education (CAE) system is a highly anticipated procedure for increasing productivity and decreasing design-to-market time. In this work, a comparative study is presented evaluating different novel methodologies and meshing techniques approaches, for automatically creating the boundary element method (BEM) computational model for stress calculations of arbitrary shaped 3D beams. The comparison covers adaptive meshing techniques. The contribution of postgraduate students was essential in testing the code created and validating the methodologies. The educational aspect of obtaining high quality insight on state-of-the-art integrated CAE procedures proved to be of high value.

Keywords: Adaptive meshing, CAD, CAE, BEM, stress analysis, 3D beam

INTRODUCTION

Studying and processing complex geometric models, using arithmetic analysis codes to obtain their mechanical properties, requires a simplification of the model as a first step towards calculations. Despite the existence of powerful CAD-geometry processing algorithms, the smooth transition to integrated stress analysis calculations continues to pose problems.

Alongside the widely used finite element method (FEM), the boundary element method (BEM) is also used to analyse constructions. Among other things, it offers the advantage of compatibility in describing the required computational geometry using the geometrical representation of the object being designed in the computer-aided design (CAD) system. Even though BEM analysis offers increased accuracy in many construction calculation problems, particularly in stress analysis, there are very few computation codes that have gone beyond research. Although BEM's direct relationship to the CAD geometry offers significant advantages, few steps have been achieved in that direction [1], resulting in minor use in industry.

It has been widely recognised that the accuracy of BEM depends on the way curves or surfaces bounding a domain are discretised. Thus, the type of elements used in the mesh, their number and their distribution are critical parameters determining the quality of the solution obtained from the analysis. Considering that best results are obtained when the boundary elements number is higher in areas where the geometry changes rapidly and areas where the solution is expected to show strong variations [2], it is crucial for the boundary to be discrete in such a way (nodes and elements) as to produce accurate results. The automated techniques that redefine the mesh are called adaptive meshing.

The aim of this work is to quantify the difference of standard versus adaptive meshing techniques that both include analytical calculated tools, thus reduce calculation error among others.

METHODOLOGY

3D CAD systems have various geometric model management philosophies. The object description method selected as the basis for this article describes an object using a constructive solid geometry (CSG) type *construction history* tree, where objects are a result of an addition, extrusion, linear and rotational operations sequence, as well as sets of operations (union, cut, intersection) in the form of parametric features (holes, threads and other construction elements). Such elements constitute an assembly described in boundary representation (B-rep) format, while curves and parametric surfaces are described using non-uniform rational b-splines (NURBS) analytical equations and splines. NURBS geometry description provides the advantage of immediate use to generate the computational model for BEM calculation. In this way, the methodology being presented can directly cooperate with the geometric cores used by the most important solid modellers of today's CAD systems.

The first step in creating the computational model is the inverse redesign of the cross-sections along the structure, in a way to be usable according to the stress analysis calculation needs. The goal of this is to obtain the maximum equivalent stress $\sigma_v(x,y)$, as well as its position P(x,y) on the boundary for every cross-section along a 3D beam, while the cross section geometry is random, as shown in Figure 1.

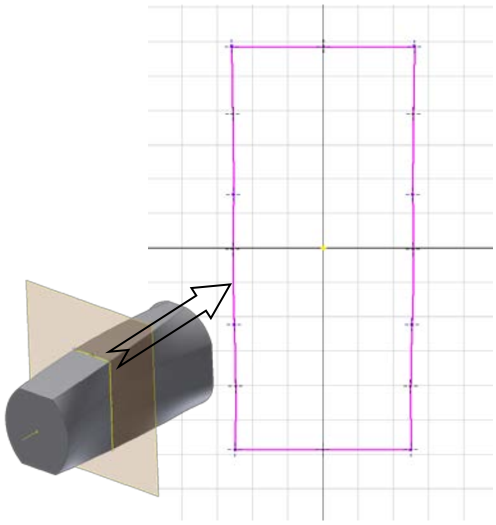

Figure 1: 2D model based on 3D sectioning.

For this calculation, the von Mises equation of combined fatigue in sidelong bending and torsion will be used. The equivalent stress for a point P(x,y) on the boundary of the cross-section is calculated using the Equation (1).

$$\sigma_v(x,y) = \sqrt{\sigma_b^{\,2}(x,y) + 3\tau_t^{\,2}(x,y)} \tag{1}$$

Where σ_b is bending and τ_t torsional stress. The bending stress is calculated based on analytical and the torsional on numerical (BEM) methods.

The distribution of torsion strains as a result of applying a torsion torque on the boundary of the cross section, should be calculated for all cross sections that are perpendicular to the beam (Figure 2), where they obtain their higher values. The distance between two consecutive cross sections can be adaptively calculated based on previous work [14][15].

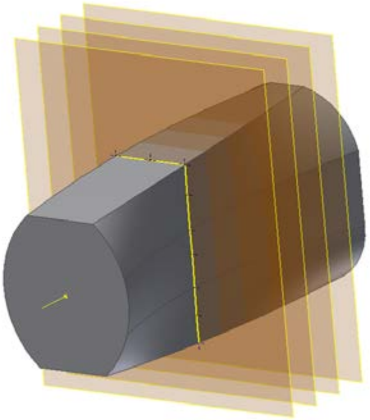

Figure 2: Parallel planes on 3D beam.

For the purpose of generating the computational model, the use of BEM requires the description of the cross-section geometry using a suitable number of nodes on its boundary. To create those nodes, an analysis is performed in the CAD system of the cross-section elements. These may be straight line parts, circle arcs, ellipse sections, splines or a combination of all of them, obtained from the CAD to get perfect fit of the beam's section on every plane.

For such geometric elements, their topology is analysed determining the starting and ending points of each element and are placed in such way to generate a closed cross section. Each CAD geometric element is described by a different number of nodes. Every such element (straight line, circle, arc, ellipse, spline, etc) is converted into its NURBS representation, inside the CAD system, so a single mathematical function of the closed cross section is used. The nodes may be equidistant amongst them, or their geometric position may be determined by a function, as Sauer suggested [13], where nodes become denser towards the edges. In this article, the number and position of the nodes that compose the mesh, are generated automatically by using two different approaches. The first approach is based on equivalent distance between nodes on each CAD element and the second is based on an adaptive procedure.

Adaptive Meshing

Every numerical method includes different type of errors, as classified by Zhao and Wang [5]. Since idealisation errors are small, discretisation errors are considered to have an important role. Idealisation errors occur due to transformation from a geometric model to a mathematical one, and discretisation errors include errors as result of the used polynomial interpolation function to describe the boundary. To evaluate the error in a BEM solution, an error indicator must be used. Guiggiani was one of the first researchers to propose the use of new error indicators especially designed for BEM, instead of using FEM ones [6]. He suggested the use of the difference between two solutions, which are obtained from the same element, doubling the number of nodes and elements each time. This difference will determine whether an element needs re-meshing or not, and through such iterations the final mesh is formed, becoming finer near areas with high error.

The methodology is based on a posteriori error indicator developed through previous works [6][7][12], but its novelty can be examined on three main characteristics. First, the integration of analytical methods within the stress calculations; second, the use of Lagrange BEM elements with such an error indicator and, finally, the implementation of the methodology within the boundaries of a modern 3D CAD solid modeller. On this point, one must note that none of the commercial BEM software has the feature of adaptive meshing or using analytical methods for stress calculations that both lead to reduced error, thus increase accuracy.

IMPLEMENTATION TESTS

In order to validate our methodology, the authors implemented a program that combined all the above features, within the boundaries of a modern CAD system, using its application programming interface (API).

Rectangular Cross Section Beam

The first set of tests was held on a rectangular cross-section beam that produced analytical results and the magnitude of the exact true error can be calculated. The input used loads are bending (M_{bx} = -560,000 N·mm and M_{by} = 280,000 N·mm) and torsion (M_t = 900,000 N·mm) moments (Figure 3). Figures 4 and 5 show the relevant stress diagrams.

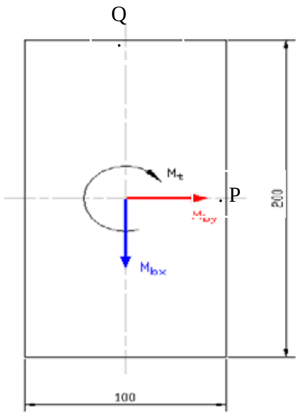

Figure 3: Moments on rectangular section.

Table 1: Results of rectangular cross section beam.

		Torsion stress			True error	
S/N	Point	Analytical	With error indicator	Without error indicator	Without error indicator	With error indicator
		(N/mm^2)	(N/mm^2)	(N/mm^2)	(%)	(%)
1	P	1.833	1.8315	1.829	0.22	0.082
2	Q	1.466	1.4650	1.457	0.62	0.071

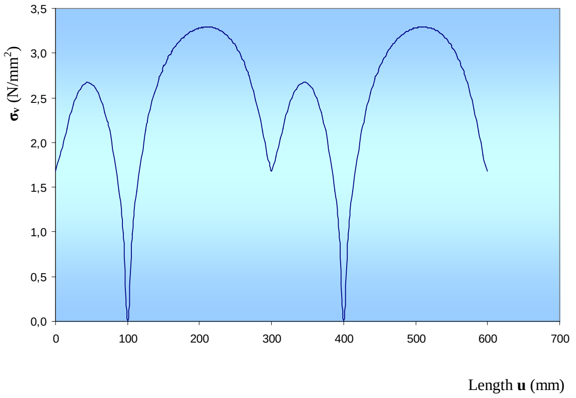

Figure 4: Diagram of equivalent stress on the boundary of rectangular section beam.

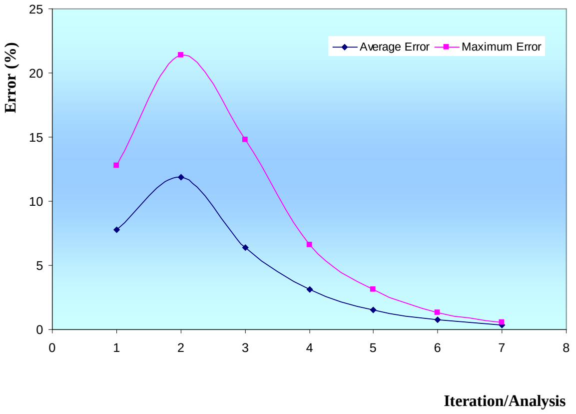

Figure 5: Convergence diagram - equivalent stress of rectangular section beam.

I-profile Cross Section Beam

Another set of tests was also conducted using an *I* profile cross section beam (Figure 6), according to DIN 1028-2. The input loads used are bending (M_{bx} = -560,000 N·mm and M_{by} = 280,000 N·mm) and torsion (M_t = 50,000 N·mm) moments. After six iterations, one manages to have every element's error lowered to less than 1%, having the maximum value 13.27 N/mm^2 [14]. Figures 7 and 8 show the relevant stress diagrams.

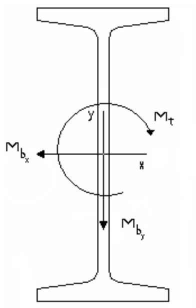

Figure 6: Moments on *I*-profile section beam.

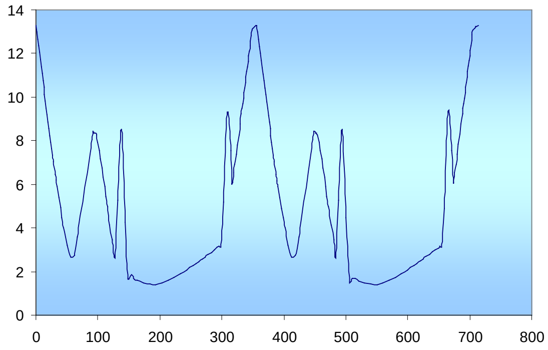

Figure 7: Diagram of equivalent stress on the boundary of *I*-profile section beam.

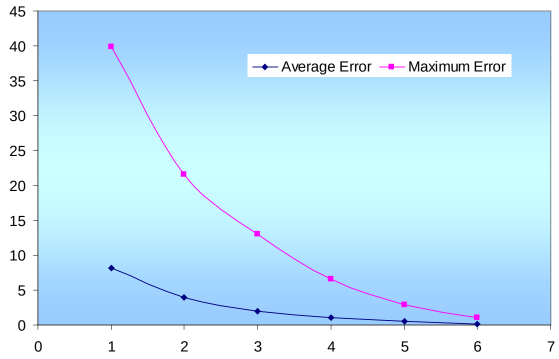

Figure 8: Convergence diagram - equivalent stress of *I*-profile section beam.

IMPACT ON ENGINEERING AND TECHNOLOGY EDUCATION

Undergraduate and postgraduate students, engineers and professors of the Mechanical Engineering Department at the Piraeus University of Applied Sciences, in collaboration with the Mechanical and Automotive Engineering Department of Kingston University, London, contributed to the completion of the presented work. Next step would be to evolve the existing methodologies and embed them with corresponding adaptive slicing procedures [15][16] to generate a complete initial version of stress calculation integrated tool based on BEM meshing techniques. This would help the students get an easy self-explaining visual feeling on how the design of mechanical parts affects its strength due to different loading and boundary conditions.

CONCLUSIONS

A comparative study of different meshing techniques and quantitative analysis was presented in this work. The objective was to explore the magnitude of difference between the presented approaches. The aim was to lower the error on such calculations by using analytical methods combined with BEM on one hand, and taking advantage of CAD's technology as much as possible on the other. This work can be further developed by combining it with other slicing procedures [15], [16] in order to obtained accurate results along beam's length or embed quality tools indicator of guiding the meshing procedure.

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BIOGRAPHIES

Vasileios D. Sagias graduated from the Mechanical Engineering Department at Piraeus University of Applied Sciences in Piraeus-Athens, Greece, in 2004. He holds a Master of Science degree in advanced industrial and manufacturing systems from Kingston University, UK, at which he is currently a PhD student. He has had 15 years' experience, mainly in the quarrying machine industry, in the fields of mechanical design, CAD (2D and 3D), CAE, CAM, CNC, CAD customisation using API, but also in production and inventory management. He started his career as a mechanical designer, which has led to his appointment as Head of Design-R/D Department. Also, for the past 10 years, he has been working as a safety engineer and is currently an Adjunct Professor at Piraeus University of Applied Sciences, Piraeus-Athens, Greece. His teaching-laboratory experience is mainly in mechanical design, CAD, CAE, CAM and production management related modules. His research areas include adaptive meshing techniques, CAD technologies corresponding with numerical methods (such as BEM and FEM) and design of experiments.

Prof. Dr-Ing. Constantinos Stergiou received his mechanical engineering degree from the National Technical University, Athens, Greece, and his PhD from the Technische Universität Darmstadt, Germany. He is a professor at Piraeus University of Applied Sciences, and is the Head of the Mechanical Engineering Department. He has been an Honorary Professor at Kingston University, London, in the Faculty of Science, Engineering and Computing since May 2013. In 2002, he organised and became the Academic Director of the MSc in Advanced Industrial and Manufacturing Systems, a collaboration with the Faculty of Science, Engineering and Computing of Kingston University, London, UK. He has professional experience in technical bureaux and research projects. He lectures in engineering design and computer-aided design, and has written eight books, most of which are the official handbooks given out to engineering students at various universities and TEI across Greece. He has organised training programmes for manufacturing plants in countries in, and outside Europe. He has had over 65 papers published in international journals and conference proceedings, is supervising four PhD students and has supervised 65 MSc theses, 25 BSc theses and 50 industrial experience semester students.

Redha Benhadj-Djilali is a chartered engineer, member of the Institution of Mechanical Engineers and member of the British Computer Society. He specialises in CAD/CAM/CAE, engineering design and manufacturing automation and has been at Kingston University since 1994. He worked in the manufacturing industry before starting his academic career at Kingston University, where he also held a research fellowship and, then, a lecturer position. His research project was based on the development of a pneumatic array of air jet proximity to tactile sensing device for manufacturing parts identification, slip detection and force monitoring. He also held a research fellowship for more than two years, and during that time, he was involved in promoting research in manufacturing automation and assembly. He also carried out further research in the application of the tactile sensing device to monitor slip and gripping force to assist robot part handling and assembly. He is on the editorial boards of three international journals: Sensor Review, Industrial Robots and Assembly Automation, and reviews other international journal and conference papers. His research interests are in the fields of design and manufacture, advanced CAD/CAM, robot tactile sensing and AI.

Petros Mostratos graduated as a mechanical engineer from Piraeus University of Applied Sciences in Piraeus-Athens, Greece, in 2015. Currently, he is a student in a Master of Science degree programme in advanced industrial and manufacturing systems class at Kingston University, London. During his Bachelor's thesis programme, he took part in research development, part of which is presented in this article. Currently, he works as a supervisor engineer in a waste oily-water treatment plant, in the Hellenic Environmental Centre, Piraeus, a member of Aegean Marine Petroleum Group of Companies, and the largest independent physical supplier in the world. The company's client catalogue includes all Greek and foreign shipping companies.